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Conservation of Angular Momentum

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ABSTRACT: The term conservation refers to something which doesn't change. Angular momentum is the rotational analogue of linear momentum. In this article, Let's study the law of conservation of angular momentum. In physics, angular momentum (sometimes called moment of momentum or rotational momentum) is the rotational analog of linear momentum. It is an important physical quantity because it is a conserved quantity – the total angular momentum of a closed system remains constant. Angular momentum has both a direction and a magnitude, and both are conserved. Bicycles and motorcycles, flying discs,[1] rifled bullets, and gyroscopes owe their useful properties to conservation of angular momentum. Conservation of angular momentum is also why hurricanes[2] form spirals and neutron stars have high rotational rates. In general, conservation limits the possible motion of a system, but it does not uniquely determine it.

The three-dimensional angular momentum for a point particle is classically represented as a pseudovector r × p, the cross product of the particle's position vector r (relative to some origin) and its momentum vector; the latter is p = mv in Newtonian mechanics. Unlike linear momentum, angular momentum depends on where this origin is chosen, since the particle's position is measured from it.

Angular momentum is an extensive quantity; that is, the total angular momentum of any composite system is the sum of the angular momenta of its constituent parts. For a continuous rigid body or a fluid, the total angular momentum is the volume integral of angular momentum density (angular momentum per unit volume in the limit as volume shrinks to zero) over the entire body.

Similar to conservation of linear momentum, where it is conserved if there is no external force, angular momentum is conserved if there is no external torque. Torque can be defined as the rate of change of angular momentum, analogous to force. The net external torque on any system is always equal to the total torque on the system; in other words, the sum of all internal torques of any system is always 0 (this is the rotational analogue of Newton's third law of motion). Therefore, for a closed system (where there is no net external torque), the total torque on the system must be 0, which means that the total angular momentum of the system is constant. The change in angular momentum for a particular interaction is called angular impulse, sometimes twirl.[3] Angular impulse is the angular analog of (linear) impulse.

KEYWORDS; angular momentum , Newtonian mechanics, Newton's third law of motion, angular impulse, Torque

I.INTRODUCTION

Just as for angular velocity, there are two special types of angular momentum of an object: the spin angular momentum is the angular momentum about the object's centre of mass, while the orbital angular momentum is the angular momentum about a chosen center of rotation. The Earth has an orbital angular momentum by nature of revolving around the Sun, and a spin angular momentum by nature of its daily rotation around the polar axis. The total angular momentum is the sum of the spin and orbital angular momenta.[1,2,3] In the case of the Earth the primary conserved quantity is the total angular momentum of the solar system because angular momentum is exchanged to a small but important extent among the planets and the Sun. The orbital angular momentum vector of a point particle is always parallel and directly proportional to its orbital angular velocity vector ω, where the constant of proportionality depends on both the mass of the particle and its distance from origin. The spin angular momentum vector of a rigid body is proportional but not always parallel to the spin angular velocity vector Ω, making the constant of proportionality a second-rank tensor rather than a scalar.

To completely define orbital angular momentum in three dimensions, it is required to know the rate at which the position vector sweeps out angle, the direction perpendicular to the instantaneous plane of angular displacement, and the mass involved, as well as how this mass is distributed in space.[9] By retaining this vector nature of angular momentum, the general nature of the equations is also retained, and can describe any sort of three-dimensional motion about the center of rotation – circular, linear, or otherwise. Angular momentum can be described as the rotational analog of linear momentum. Like linear momentum it involves elements of mass and displacement. Unlike linear momentum it also involves elements of position and shape.

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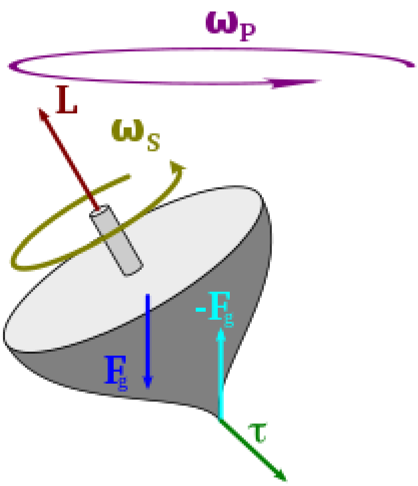
Many problems in physics involve matter in motion about some certain point in space, be it in actual rotation about it, or simply moving past it, where it is desired to know what effect the moving matter has on the point—can it exert energy upon it or perform work about it? Energy, the ability to do work, can be stored in matter by setting it in motion—a combination of its inertia and its displacement. Inertia is measured by its mass, and displacement by its velocity. Their product, is the matter's momentum.[10] Referring this momentum to a central point introduces a complication: the momentum is not applied to the point directly. For instance, a particle of matter at the outer edge of a wheel is, in effect, at the end of a lever of the same length as the wheel's radius, its momentum turning the lever about the center point. This imaginary lever is known as the moment arm. It has the effect of multiplying the momentum's effort in proportion to its length, an effect known as a moment. Hence, the particle's momentum referred to a particular point.

Angular momentum's dependence on position and shape is reflected in its units versus linear momentum: kg·m^2/s or N·m·s for angular momentum versus kg·m/s or N·s for linear momentum. When calculating angular momentum as the product of the moment of inertia times the angular velocity, the angular velocity must be expressed in radians per second, where the radian assumes the dimensionless value of unity.[5,,7,8] (When performing dimensional analysis, it may be productive to use orientational analysis which treats radians as a base unit, but this is not done in the International system of units). The units if angular momentum can be interpreted as torque·time. An object with angular momentum of L N·m·s can be reduced to zero angular velocity by an angular impulse of L N·m·s.[16][17]

The plane perpendicular to the axis of angular momentum and passing through the center of mass[18] is sometimes called the invariable plane, because the direction of the axis remains fixed if only the interactions of the bodies within the system, free from outside influences, are considered.[19] One such plane is the invariable plane of the Solar System.

For a planet, angular momentum is distributed between the spin of the planet and its revolution in its orbit, and these are often exchanged by various mechanisms. The conservation of angular momentum in the Earth–Moon system results in the transfer of angular momentum from Earth to Moon, due to tidal torque the Moon exerts on the Earth. This in turn results in the slowing down of the rotation rate of Earth, at about 65.7 nanoseconds per day,[23] and in gradual increase of the radius of Moon's orbit, at about 3.82 centimeters per year.[24]



The torque caused by the two opposing forces F_g and $-F_g$ causes a change in the angular momentum L in the direction of that torque (since torque is the time derivative of angular momentum). This causes the top to precess.

The conservation of angular momentum explains the angular acceleration of an ice skater as they bring their arms and legs close to the vertical axis of rotation. By bringing part of the mass of their body closer to the axis, they decrease their body's moment of inertia. Because angular momentum is the product of moment of inertia and angular velocity, if the angular momentum remains constant (is conserved), then the angular velocity (rotational speed) of the skater must increase.[10,11,12]

The same phenomenon results in extremely fast spin of compact stars (like white dwarfs, neutron stars and black holes) when they are formed out of much larger and slower rotating stars.

Conservation is not always a full explanation for the dynamics of a system but is a key constraint. For example, a spinning top is subject to gravitational torque making it lean over and change the angular momentum about

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the nutation axis, but neglecting friction at the point of spinning contact, it has a conserved angular momentum about its spinning axis, and another about its precession axis. Also, in any planetary system, the planets, star(s), comets, and asteroids can all move in numerous complicated ways, but only so that the angular momentum of the system is conserved.

Noether's theorem states that every conservation law is associated with a symmetry (invariant) of the underlying physics. The symmetry associated with conservation of angular momentum is rotational invariance. The fact that the physics of a system is unchanged if it is rotated by any angle about an axis implies that angular momentum is conserved.[25]

While angular momentum total conservation can be understood separately from Newton's laws of motion as stemming from Noether's theorem in systems symmetric under rotations, it can also be understood simply as an efficient method of calculation of results that can also be otherwise arrived at directly from Newton's second law, together with laws governing the forces of nature (such as Newton's third law, Maxwell's equations and Lorentz force). Indeed, given initial conditions of position and velocity for every point, and the forces at such a condition, one may use Newton's second law to calculate the second derivative of position, and solving for this gives full information on the development of the physical system with time.[26] Note, however, that this is no longer true in quantum mechanics, due to the existence of particle spin, which is angular momentum that cannot be described by the cumulative effect of point-like motions in space.

II.DISCUSSION

In quantum mechanics, angular momentum (like other quantities) is expressed as an operator, and its one-dimensional projections have quantized eigenvalues. Angular momentum is subject to the Heisenberg uncertainty principle, implying that at any time, only one projection (also called "component") can be measured with definite precision; the other two then remain uncertain. Because of this, the axis of rotation of a quantum particle is undefined. Quantum particles do possess a type of non-orbital angular momentum called "spin", but this angular momentum does not correspond to a spinning motion.[32] In relativistic quantum mechanics the above relativistic definition becomes a tensorial operator. Tropical cyclones and other related weather phenomena involve conservation of angular momentum in order to explain the dynamics. Winds revolve slowly around low pressure systems, mainly due to the coriolis effect. If the low pressure intensifies and the slowly circulating air is drawn toward the center, the molecules must speed up in order to conserve angular momentum. By the time they reach the center, the speeds become destructive.[2]

Johannes Kepler determined the laws of planetary motion without knowledge of conservation of momentum. However, not long after his discovery their derivation was determined from conservation of angular momentum. Planets move more slowly the further they are out in their elliptical orbits, which is explained intuitively by the fact that orbital angular momentum is proportional to the radius of the orbit. Since the mass does not change and the angular momentum is conserved, the velocity drops.[1,17,18]

Tidal acceleration is an effect of the tidal forces between an orbiting natural satellite (e.g. the Moon) and the primary planet that it orbits (e.g. Earth). The gravitational torque between the Moon and the tidal bulge of Earth causes the Moon to be constantly promoted to a slightly higher orbit and Earth to be decelerated in its rotation. The Earth loses angular momentum which is transferred to the Moon such that the overall angular momentum is conserved.

Examples of using conservation of angular momentum for practical advantage are abundant. In engines such as steam engines or internal combustion engines, a flywheel is needed to efficiently convert the lateral motion of the pistons to rotational motion.

Inertial navigation systems explicitly use the fact that angular momentum is conserved with respect to the inertial frame of space. Inertial navigation is what enables submarine trips under the polar ice cap, but are also crucial to all forms of modern navigation.

Rifled bullets use the stability provided by conservation of angular momentum to be more true in their trajectory. The invention of rifled firearms and cannons gave their users significant strategic advantage in battle, and thus were a technological turning point in history.

Isaac Newton, in the Principia, hinted at angular momentum in his examples of the first law of motion,

A top, whose parts by their cohesion are perpetually drawn aside from rectilinear motions, does not cease its rotation, otherwise than as it is retarded by the air. The greater bodies of the planets and comets, meeting with less resistance in more free spaces, preserve their motions both progressive and circular for a much longer time.[43]

He did not further investigate angular momentum directly in the Principia, saying:

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From such kind of reflexions also sometimes arise the circular motions of bodies about their own centres. But these are cases which I do not consider in what follows; and it would be too tedious to demonstrate every particular that relates to this subject.[44]

However, his geometric proof of the law of areas is an outstanding example of Newton's genius, and indirectly proves angular momentum conservation in the case of a central force.[20,21,22]

III.RESULTS

Leonhard Euler, Daniel Bernoulli, and Patrick d'Arcy all understood angular momentum in terms of conservation of areal velocity, a result of their analysis of Kepler's second law of planetary motion. It is unlikely that they realized the implications for ordinary rotating matter.[45]

In 1736 Euler, like Newton, touched on some of the equations of angular momentum in his Mechanica without further developing them.[46]

Bernoulli wrote in a 1744 letter of a "moment of rotational motion", possibly the first conception of angular momentum as we now understand it.[47]

In 1799, Pierre-Simon Laplace first realized that a fixed plane was associated with rotation—his invariable plane.

Louis Poinsot in 1803 began representing rotations as a line segment perpendicular to the rotation, and elaborated on the "conservation of moments".[23,2,25]

In 1852 Léon Foucault used a gyroscope in an experiment to display the Earth's rotation.

William J. M. Rankine's 1858 Manual of Applied Mechanics defined angular momentum in the modern sense for the first time:

...a line whose length is proportional to the magnitude of the angular momentum, and whose direction is perpendicular to the plane of motion of the body and of the fixed point, and such, that when the motion of the body is viewed from the extremity of the line, the radius-vector of the body seems to have right-handed rotation.

In an 1872 edition of the same book, Rankine stated that "The term angular momentum was introduced by Mr. Hayward,"[48] probably referring to R.B. Hayward's article On a Direct Method of estimating Velocities, Accelerations, and all similar Quantities with respect to Axes moveable in any manner in Space with Applications,[49] which was introduced in 1856, and published in 1864. Rankine was mistaken, as numerous publications feature the term starting in the late 18th to early 19th centuries.[50] However, Hayward's article apparently was the first use of the term and the concept seen by much of the English-speaking world. Before this, angular momentum was typically referred to as "momentum of rotation" in English.[51]

The angular momentum of light is a vector quantity that expresses the amount of dynamical rotation present in the electromagnetic field of the light. While traveling approximately in a straight line, a beam of light can also be rotating (or "spinning", or "twisting") around its own axis. This rotation, while not visible to the naked eye, can be revealed by the interaction of the light beam with matter.

There are two distinct forms of rotation of a light beam, one involving its polarization and the other its wavefront shape. These two forms of rotation are therefore associated with two distinct forms of angular momentum, respectively named light spin angular momentum (SAM) and light orbital angular momentum (OAM).

The total angular momentum of light (or, more generally, of the electromagnetic field and the other force fields) and matter is conserved in time.

Light, or more generally an electromagnetic wave, carries not only energy but also momentum, which is a characteristic property of all objects in translational motion. The existence of this momentum becomes apparent in the "radiation pressure" phenomenon, in which a light beam transfers its momentum to an absorbing or scattering object, generating a mechanical pressure on it in the process.

Light may also carry angular momentum, which is a property of all objects in rotational motion. For example, a light beam can be rotating around its own axis while it propagates forward. Again, the existence of this angular momentum can be made evident by transferring it to small absorbing or scattering particles, which are thus subject to an optical torque.

For a light beam, one can usually distinguish two "forms of rotation", the first associated with the dynamical rotation of the electric and magnetic fields around the propagation direction, and the second with the dynamical rotation of light rays around the main beam axis. These two rotations are associated with two forms of angular momentum, namely

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SAM and OAM. However this distinction becomes blurred for strongly focused or diverging beams, and in the general case only the total angular momentum of a light field can be defined. An important limiting case in which the distinction is instead clear and unambiguous is that of a "paraxial" light beam, that is a well collimated beam in which all light rays (or, more precisely, all Fourier components of the optical field) only form small angles with the beam axis.

For such a beam, SAM is strictly related with the optical polarization, and in particular with the so-called circular polarization. OAM is related with the spatial field distribution, and in particular with the wavefront helical shape.

In addition to these two terms, if the origin of coordinates is located outside the beam axis, there is a third angular momentum contribution obtained as the cross-product of the beam position and its total momentum. This third term is also called "orbital", because it depends on the spatial distribution of the field. However, since its value is dependent from the choice of the origin, it is termed "external" orbital angular momentum, as opposed to the "internal" OAM appearing for helical beams.[28]

When a light beam carrying nonzero angular momentum impinges on an absorbing particle, its angular momentum can be transferred on the particle, thus setting it in rotational motion. This occurs both with SAM and OAM. However, if the particle is not at the beam center the two angular momenta will give rise to different kinds of rotation of the particle. SAM will give rise to a rotation of the particle around its own center, i.e., to a particle spinning. OAM, instead, will generate a revolution of the particle around the beam axis.[3][4][5] .

In the case of transparent media, in the paraxial limit, the optical SAM is mainly exchanged with anisotropic systems, for example birefringent crystals. Indeed, thin slabs of birefringent crystals are commonly used to manipulate the light polarization. Whenever the polarization ellipticity is changed, in the process, there is an exchange of SAM between light and the crystal. If the crystal is free to rotate, it will do so. Otherwise, the SAM is finally transferred to the holder and to the Earth.

IV.CONCLUSIONS

The applications of the spin angular momentum of light are undistinguishable from the innumerable applications of the light polarization and will not be discussed here.[28] The possible applications of the orbital angular momentum of light are instead currently the subject of research. In particular, the following applications have been already demonstrated in research laboratories, although they have not yet reached the stage of commercialization:

1. Orientational manipulation of particles or particle aggregates in optical tweezers[16]
2. High-bandwidth information encoding in free-space optical communication[17]
3. Higher-dimensional quantum information encoding, for possible future quantum cryptography or quantum computation applications[18][19][20]
4. Sensitive optical detection[21]

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