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Mathematical Optimization in Network Design

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ABSTRACT: Mathematical Optimization plays a pivotal role in the design and management of modern networks, enabling efficient allocation of resources and robust performance under varying conditions. This paper explores key optimization models and algorithms used in network design, focusing on maximizing throughput, minimizing latency, and ensuring resilience. We discuss various optimization techniques such as linear programming, integer programming, and heuristic methods tailored to different network types including telecommunications, transportation, and computer networks. Real-world applications and case studies demonstrate the effectiveness of these approaches in enhancing network efficiency and reliability.

KEYWORDS: Mathematical Optimization, Network Design, Linear Programming, Integer Programming,

I. INTRODUCTION

Mathematical optimization in network design is a pivotal methodology used to determine the most efficient configuration of network elements, such as nodes and edges, to meet specified objectives and constraints. This field blends principles from mathematics, computer science, and engineering to solve complex problems in telecommunications, transportation, logistics, and other areas requiring robust network infrastructures. The primary goal of network design optimization is to achieve optimal performance, which might involve minimizing costs, maximizing reliability, or balancing loads within the network. By defining decision variables, objective functions, and constraints, these problems can be formulated mathematically. For instance, variables might represent the presence of links between nodes, while the objective function could aim to minimize the total cost of these links or maximize network throughput. Constraints ensure the feasibility of the solution, addressing issues such as flow conservation, capacity limits, and connectivity requirements. Advanced optimization solvers, including CPLEX and Gurobi, or open-source tools like PuLP and OR-Tools, are employed to find solutions to these complex models. Mathematical optimization in network design is essential for creating efficient, reliable, and cost-effective networks. It provides a systematic approach to addressing practical challenges, ensuring that the final network configuration meets operational demands and resource constraints effectively [1].

II. FUNDAMENTALS OF MATHEMATICAL OPTIMIZATION

Mathematical optimization is a fundamental discipline in applied mathematics and operations research that focuses on finding the best solution from a set of feasible solutions for a given problem. This involves the maximization or minimization of an objective function, which quantifies the goal of the optimization. The process starts by defining decision variables, which represent the choices available in the problem. These variables are subject to constraints that model the limitations or requirements of the real-world scenario. There are several types of optimization problems, including linear, non-linear, integer, and mixed-integer programming. In linear programming (LP), both the objective function and the constraints are linear. An example is minimizing the cost of production subject to resource availability constraints. Non-linear programming (NLP) deals with problems where the objective function or constraints are non-linear, making them more complex and requiring specialized solution techniques [2-3].

Integer programming (IP) restricts some or all decision variables to be integers, which is common in problems where decisions are yes/no (binary) or countable quantities. Mixed-integer programming (MIP) combines continuous and integer variables, providing a versatile framework for many practical problems.

Key concepts in optimization include feasible region, objective function, and optimal solution. The feasible region is the set of all possible solutions that satisfy the constraints. The objective function evaluates the quality of each solution, and the optimal solution is the one that achieves the best value for the objective function within the feasible region.

Optimization problems can be solved using various methods. Exact algorithms, such as the Simplex method for LP or branch-and-bound for IP, guarantee finding an optimal solution. Heuristic and metaheuristic methods, like genetic algorithms and simulated annealing, provide good solutions within reasonable time frames, especially for large or complex problems where exact methods are computationally infeasible.



In summary, mathematical optimization is a powerful tool for decision-making, enabling the systematic evaluation and selection of the best option among many, under given constraints. It is widely applied in diverse fields such as economics, engineering, logistics, and network design, driving efficiency and effectiveness in complex systems [4].

III. LITERATURE REVIEW

Vasan & Simonovic (2010). The paper describes the development of a DENET computer model that involves the application of an evolutionary optimization technique, differential evolution, linked to the hydraulic simulation solver, EPANET, for optimal design of water distribution networks. A model is formulated with the objective of minimizing cost and this formulation is applied to two benchmark water distribution system optimization problems—New York water supply system and Hanoi water distribution network. The study yielded promising results as compared with earlier studies in the literature and encouraged to reformulate the model for a new objective of maximizing network resilience. The results of the analysis demonstrate that DENET can be considered as a potential alternative tool for economical and reliable water distribution network planning and management.

Pathak & Dutta (2010). Over the last decade, the paradigm of Wireless Mesh Networks (WMNs) has matured to a reasonably commonly understood one, and there has been extensive research on various areas related to WMNs such as design, deployment, protocols, performance, etc. The quantity of research being conducted in the area of wireless mesh design has dramatically increased in the past few years, due to increasing interest in this paradigm as its potential for the "last few miles", and the possibility of significant wireless services in metropolitan area networks. This recent work has focused increasingly on joint design problems, together with studies in designing specific aspects of the WMN such as routing, power control etc. in isolation. While excellent surveys and tutorials pertaining to WMNs exist in literature, the explosive growth of research in the area of specific design issues, and especially joint design, has left them behind. Our objective in this paper is to identify the fundamental WMN design problems of interference modelling, power control, topology control, link scheduling, and routing, and provide brief overviews, together with a survey of the recent research on these topics, with special stress on joint design methods. We believe this paper will fulfill an outstanding need in informing the interested student and researcher in getting familiar with this abundant recent research area, and starting research.

Wang & Kuo (2012). In heterogeneous wireless networks, an important task for mobile terminals is to select the best network for various communications at any time anywhere, usually called network selection. In recent years, this topic has been widely studied by using various mathematical theories. The employed theory decides the objective of optimization, complexity and performance, so it is a must to understand the potential mathematical theories and choose the appropriate one for obtaining the best result. Therefore, this paper systematically studies the most important mathematical theories used for modelling the network selection problem in the literature. With a carefully designed unified scenario, we compare the schemes of various mathematical theories and discuss the ways to benefit from combining multiple of them together. Furthermore, an integrated scheme using multiple attribute decision making as the core of the selection procedure is proposed.

Saka & Geem (2013). The type of mathematical modelling selected for the optimum design problems of steel skeletal frames affects the size and mathematical complexity of the programming problem obtained. Survey on the structural optimization literature reveals that there are basically two types of design optimization formulation. In the first type only cross-sectional properties of frame members are taken as design variables. In such formulation when the values of design variables change during design cycles, it becomes necessary to analyse the structure and update the response of steel frame to the external loading. Structural analysis in this type is a complementary part of the design process. In the second type joint coordinates are also treated as design variables in addition to the cross-sectional properties of members. Such formulation eliminates the necessity of carrying out structural analysis in every design cycle. The values of the joint displacements are determined by the optimization techniques in addition to cross sectional properties. The structural optimization literature contains structural design algorithms that make use of both type of formulation. In this study a review is carried out on mathematical and metaheuristic algorithms where the effect of the mathematical modelling on the efficiency of these algorithms is discussed.

Ogryczak et al. (2014). Optimization models related to designing and operating complex systems are mainly focused on some efficiency metrics such as response time, queue length, throughput, and cost. However, in systems which serve many entities there is also a need for respecting fairness: each system entity ought to be provided with an adequate share of the system's services. Still, due to system operations-dependant constraints, fair treatment of the entities does not directly imply that each of them is assigned equal amount of the services. That leads to concepts of fair optimization expressed by the equitable models that represent inequality averse optimization rather than strict inequality minimization; a particular widely applied example of that concept is the so-called lexicographic maximin

optimization (max-min fairness). The fair optimization methodology delivers a variety of techniques to generate fair and efficient solutions. This paper reviews fair optimization models and methods applied to systems that are based on some kind of network of connections and dependencies, especially, fair optimization methods for the location problems and for the resource allocation problems in communication networks.

Lazaropoulos (2014). This paper introduces the broadband over power lines-enhanced network model (BPLeNM) that is suitable for efficiently delivering the generated data of wireless sensor networks (WSNs) of overhead high-voltage (HV) power grids to the substations. BPLeNM exploits the high data rates of the already installed BPL networks across overhead HV grids. BPLeNM is compared against other two well-verified network models from the relevant literature: the linear network model (LNM) and the optimal arrangement network model (OANM). The contribution of this paper is threefold. First, the general mathematical framework that is necessary for describing WSNs of overhead HV grids is first presented. In detail, the general mathematical formulation of BPLeNM is proposed while the existing formulations of LNM and OANM are extended so as to deal with the general case of overhead HV grids. Based on these general mathematical formulations, the general expression of maximum delay time of the WSN data is determined for the three network models. Second, the three network models are studied and assessed for a plethora of case scenarios. Through these case scenarios, the impact of different lengths of overhead HV grids, different network arrangements, new communications technologies, variation of WSN density across overhead HV grids, and changes of generated WSN data rate on the maximum delay time is thoroughly examined. Third, to assess the performance and the feasibility of the previous network models, the feasibility probability (FP) is proposed. FP is a macroscopic metric that estimates how much practical and economically feasible is the selection of one of the previous three network models. The main conclusion of this paper is that BPLeNM defines a powerful, convenient, and schedulable network model for today's and future's overhead HV grids in the smart grid (SG) landscape.

Shames & Summers (2015). We consider the problem of constructing networks that exhibit desirable algebraic rigidity properties, which can provide significant performance improvements for associated formation shape control and localization tasks. We show that the network design problem can be formulated as a submodular set function optimization problem and propose greedy algorithms that achieve global optimality or an established near-optimality guarantee. We also consider the separate but related problem of selecting anchors for sensor network localization to optimize a metric of the error in the localization solutions. We show that an interesting metric is a modular set function, which allows a globally optimal selection to be obtained using a simple greedy algorithm. The results are illustrated via numerical examples, and we show that the methods scale to problems well beyond the capabilities of current state-of-the-art convex relaxation techniques.

Yuan et al. (2016). Natural disasters such as Hurricane Sandy can seriously disrupt the power grids. To increase the resilience of an electric distribution system against natural disasters, this paper proposes a resilient distribution network planning problem (RDNP) to coordinate the hardening and distributed generation resource allocation with the objective of minimizing the system damage. The problem is formulated as a two-stage robust optimization model. Hardening and distributed generation resource placement are considered in the distribution network planning. A multi-stage and multi-zone-based uncertainty set is designed to capture the spatial and temporal dynamics of an uncertain natural disaster as an extension to the traditional N-K contingency approach. The optimal solution of the RDNP yields a resilient distribution system against natural disasters. Our computational studies on the IEEE distribution test systems validate the effectiveness of the proposed model and reveal that distributed generation is critical in increasing the resilience of a distribution system against natural disasters in the form of microgrids.

Park et. al. (2017). Wireless networked control systems (WNCSs) are composed of spatially distributed sensors, actuators, and controllers communicating through wireless networks instead of conventional point-to-point wired connections. Due to their main benefits in the reduction of deployment and maintenance costs, large flexibility and possible enhancement of safety, WNCS are becoming a fundamental infrastructure technology for critical control systems in automotive electrical systems, avionics control systems, building management systems, and industrial automation systems. The main challenge in WNCS is to jointly design the communication and control systems considering their tight interaction to improve the control performance and the network lifetime. In this survey, we make an exhaustive review of the literature on wireless network design and optimization for WNCS. First, we discuss what we call the critical interactive variables including sampling period, message delay, message dropout, and network energy consumption. The mutual effects of these communication and control variables motivate their joint tuning. We discuss the analysis and design of control systems taking into account the effect of the interactive variables on the control system performance. Moreover, we discuss the effect of controllable wireless network parameters at all layers of the communication protocols on the probability distribution of these interactive variables. We also review the current wireless network standardization for WNCS and their corresponding methodology for adapting the network parameters.



Finally, we present the state-of-the-art wireless network design and optimization for WNCS, while highlighting the trade-off between the achievable performance and complexity of various approaches. We conclude the survey by highlighting major research issues and identifying future research directions.

Heidari-Fathian, H., & Pasandideh, S. H. R. (2018). The aim of this research is to consider the issue of sustainability in designing a blood supply chain network by presenting a multi objective mixed integer mathematical programming model that aims to simultaneously minimize the total cost of the supply chain network and the total environmental impacts of the activities of the supply chain network. As the nature of supplying the blood by the donors and also demand for the blood product are uncertain, a robust optimization approach is applied in the model to deal with this type of uncertainty.

Duque et.al., (2020). This paper proposes an iterative mathematical optimization framework to solve the layout and hydraulic design problems of sewer networks. The layout selection model determines the flow rate and direction per pipe using mixed-integer programming, which results in a tree-like structured network. This network layout parametrizes a second model that determines hydraulic features including the diameter and the upstream and downstream invert elevations of pipes using a shortest path algorithm.

IV. METHODOLOGY FOR NETWORK DESIGN OPTIMIZATION

Mathematical optimization in network design typically involves formulating a problem to find the best configuration of network elements (nodes and edges) under certain constraints and objectives. Following's a general framework for modelling such problems using equations:

Basic Elements in Network Design Optimization

Nodes and Edges

Let N denote the set of nodes in the network.

Let E denote the set of edges (connections) between nodes.

Variables:

Decision variables are defined to represent the design choices. For example:

x_{ij} : Binary variable indicating if there is a link (edge) between node i and node j .

y_i : Binary variable indicating if node i is included in the network.

Objective Function

The objective function $f(x,y)$ represents what is to be optimized. It could be:

Minimizing cost:

$$\text{Minimize } \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

Maximizing performance:

$$\text{Maximize } \sum_{i \in N} u_i y_i$$

Balancing load:

$$\sum_{i \in N} \sum_{j \in N} d_{ij} x_{ij}$$

Constraints

Constraints ensure the solution meets practical or theoretical requirements. Examples include:

Flow conservation:

$$\sum_{j \in N} x_{ij} - \sum_{j \in N} x_{ji} = b_i$$

Capacity constraints:

$$\sum_{i \in N} x_{ij} \leq c_j$$

Node connectivity:

$$\sum_{j \in N} x_{ij} \geq 1$$



Formulation Examples

Minimization of Link Costs

$$\text{Minimize } \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

subject to:

$$\sum_{j \in N} x_{ij} - \sum_{i \in N} x_{ji} = b_i \quad \forall i \in N$$

$$\sum_{i \in N} x_{ij} \leq c_j \quad \forall j \in N$$

$$x_{ij} \in \{0, 1\}$$

Maximization of Network Reliability

$$\text{Maximize } \sum_{i \in N} u_i y_i$$

subject to:

$$\sum_{i \in N} x_{ij} \geq 1 \quad \forall j \in N$$

$$y_i \leq \sum_{j \in N} x_{ij} \quad \forall i \in N$$

$$y_i \in \{0, 1\}, \quad x_{ij} \in \{0, 1\}$$

Solver and Implementation

Solver: Use optimization solvers such as CPLEX, Gurobi, or open-source libraries like PuLP (Python) or OR-Tools (Google's operations research tools).

Implementation: Write code to input your problem's parameters (costs, capacities, constraints) into the solver's API or modelling language (e.g., AMPL, AIMMS, MATLAB).

Mathematical optimization provides a rigorous framework for tackling network design problems, balancing various objectives and constraints. The key lies in formulating the problem accurately with variables, objective functions, and constraints that reflect the real-world requirements of the network [5-6].

V. FORMULATION OF NETWORK DESIGN PROBLEMS

Network Models and Components

Network models and components constitute the fundamental building blocks of network design and optimization across various domains such as telecommunications, transportation, and logistics. These elements provide the structural framework necessary for designing efficient and effective networks that can meet operational requirements and optimize resource utilization.

Network Topologies define the arrangement of nodes (or vertices) and edges (or links) within a network. Common topologies include bus, star, ring, mesh, and tree structures, each offering distinct advantages in terms of scalability, reliability, and efficiency. For example, star topologies centralize communication through a hub, while mesh topologies provide redundancy with multiple interconnected paths.

Nodes and Edges are essential components within network models. Nodes represent entities like computers, routers, or cities, while edges denote the connections between them. Nodes possess properties such as degree (number of connections) and centrality (importance within the network), crucial for understanding network dynamics and performance.

Types of Networks encompass various applications

- **Communication Networks** (e.g., the Internet) prioritize bandwidth and latency management.
- **Transportation Networks** (e.g., road or rail systems) focus on optimizing routes and minimizing congestion.
- **Logistics Networks** (e.g., supply chains) emphasize efficient distribution and inventory management. Each type requires tailored design approaches to meet specific operational goals.



Graph Theory Basics underpin network modelling with mathematical rigor. A graph $G = (N, E)$ consists of nodes N and edges E , facilitating the study of paths, cycles, connectivity, and subgraphs. These concepts provide a structured way to analyze network connectivity and performance metrics.

Network Metrics and Performance metrics such as latency, throughput, and reliability are critical for assessing network efficiency and effectiveness. Latency measures the time taken for data to traverse the network, throughput denotes data transfer rates, and reliability assesses the network's robustness against failures.

Case Study Examples illustrate practical applications

- **Telecommunication Networks:** Designing resilient cellular networks to ensure continuous connectivity.
- **Transportation Networks:** Optimizing public transit routes for minimal travel times and efficient service delivery.
- **Logistics Networks:** Streamlining supply chain operations to reduce costs and enhance delivery speed.

Understanding of network models and components is essential for designing networks that meet complex operational demands across diverse domains. By leveraging graph theory principles and considering specific metrics and performance goals, network designers can create robust and efficient systems that support modern communication, transportation, and logistics infrastructures [6-7].

VI. ALGORITHMS FOR NETWORK OPTIMIZATION

Algorithms for network optimization play a critical role in designing and managing efficient network infrastructures across various domains. These algorithms are essential for solving complex optimization problems that involve determining the best configuration of nodes and edges to achieve specific objectives under constraints.

Exact Algorithms such as the **Simplex method** are foundational in linear programming (LP) and are effective for solving problems where both the objective function and constraints are linear. In network optimization, these algorithms are used, for example, to minimize costs or maximize flows while respecting capacity constraints in transportation or communication networks. **Interior point methods** are also used in LP, providing an alternative approach to solving large-scale problems efficiently.

Integer Programming (IP) algorithms extend the capabilities of LP by restricting decision variables to integer values, suitable for problems where decisions are binary (e.g., whether to include a link in a network) or require discrete choices (e.g., selecting routes in logistics networks). **Branch-and-bound** and **branch-and-cut** algorithms are widely used in IP, exploring feasible solutions systematically while pruning branches that cannot lead to better solutions, thereby optimizing the search process.

Heuristic and Metaheuristic Algorithms offer solutions for problems where exact methods are computationally prohibitive due to complexity or scale.

Greedy algorithms make locally optimal choices at each step, aiming to find a good solution quickly but without guaranteeing optimality.

Genetic algorithms, simulated annealing, and tabu search are examples of metaheuristic algorithms that explore large solution spaces by simulating biological evolution or physical processes, effectively balancing exploration and exploitation to find near-optimal solutions.

Network Flow Algorithms like the **Ford-Fulkerson method** and **Edmonds-Karp algorithm** are specialized algorithms for solving flow network problems, determining the maximum flow between nodes while respecting capacity constraints. These algorithms are crucial in optimizing traffic flow in transportation networks or data transmission in communication networks.

Algorithms for network optimization range from exact methods for solving linear and integer programming problems to heuristic and metaheuristic approaches for tackling large-scale and complex optimization challenges. Their application ensures the efficient design and management of network infrastructures that meet performance goals and operational constraints in diverse domains [8-9].

VII. CONCLUSION AND FUTURE WORK

Mathematical optimization plays a crucial role in network design, aiming to achieve optimal configurations that balance efficiency, cost-effectiveness, and performance. The conclusion of studies in this field often highlights several key findings and suggests avenues for future research and application. This research in mathematical optimization for network design has demonstrated significant advancements in developing models that can handle large-scale networks



efficiently. Techniques such as linear programming, integer programming, and heuristic approaches have been successfully applied to minimize costs, maximize throughput, and ensure robustness in network architectures. The integration of optimization with emerging technologies like machine learning and AI promises even more sophisticated network designs. These advancements enable networks to adapt dynamically to changing conditions, optimize resource allocation in real-time, and enhance overall reliability and scalability. Moreover, the future of optimization in network design will likely focus on several fronts. One crucial aspect is the incorporation of sustainability metrics, such as energy efficiency and environmental impact, into optimization models. This aligns with global efforts towards greener technologies and sustainable development. Addressing the challenges posed by the proliferation of IoT devices and the demand for ultra-low latency networks will require novel optimization approaches. These include decentralized optimization techniques and algorithms that can handle the complexity and scale of modern interconnected systems. Interdisciplinary collaboration between optimization experts, network engineers, and policymakers will be essential. This collaboration can drive the development of regulatory frameworks that encourage efficient network design and deployment. While mathematical optimization has made significant strides in network design, the field continues to evolve rapidly. Future work will focus on sustainability, resilience, and adaptability to meet the growing demands of interconnected systems in a rapidly changing technological landscape. With embracing these challenges, researchers and practitioners can pave the way for more efficient, reliable, and sustainable network infrastructures.

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