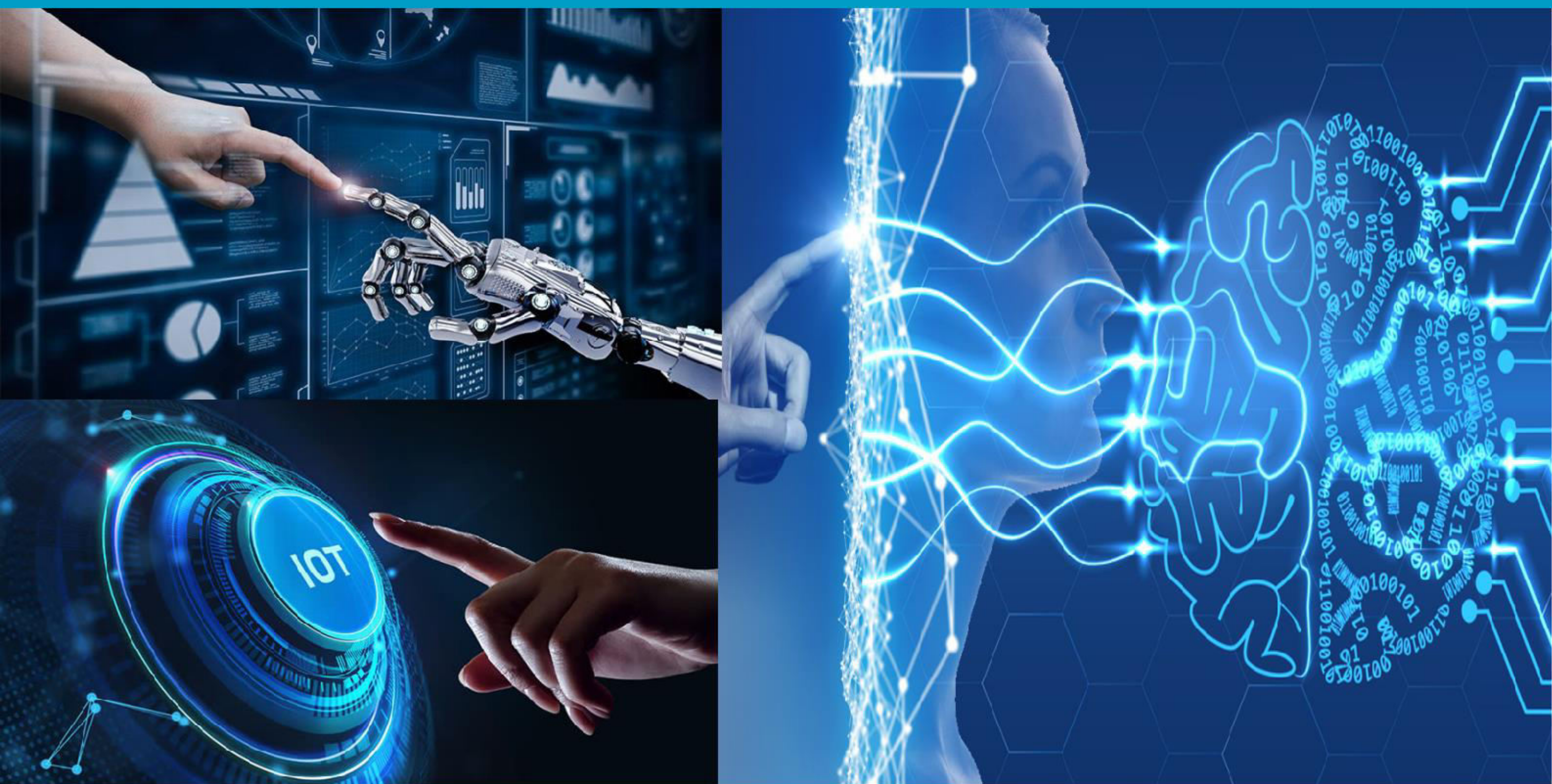
IJMRSETM

e-ISSN: 2395 - 7639



INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY RESEARCH

IN SCIENCE, ENGINEERING, TECHNOLOGY AND MANAGEMENT

Volume 10, Issue 12, December 2023

INTERNATIONAL STANDARD SERIAL NUMBER INDIA

Impact Factor: 7.580

+91 99405 72462 +91 63819 07438 ijmrsetm@gmail.com www.ijmrsetm.com

International Journal of Multidisciplinary Research in Science, Engineering, Technology & Management (IJMRSETM)

| ISSN: 2395-7639 | www.ijmrsetm.com | Impact Factor: 7.580| A Monthly Double-Blind Peer Reviewed Journal |

| Volume 10, Issue 12, December 2023 |

Multi-Objective Inventory Models In a Fuzzy Environment

Vinita Rani[1], Sohan Tyagi[2]

Sai Nath University, Jirawar, Chandway-Kuchu Road, Ormanihi, Rnchi, Jharkhand, India[1]

SRM University, Delhi-NCR Campus, Delhi Meerut Road, Modinagar, Ghaziabad, Uttar Pradesh, India[2]

ABSTRACT: This paper focuses on developing multi-item inventory models in a fuzzy environment, particularly for deteriorating items with stock-dependent demand. Its main goals are to maximize profit and minimize wastage costs, both of which are uncertain. The paper uses fuzzy linear membership functions and triangular fuzzy numbers to represent this uncertainty in objectives and inventory-related parameters. The models are solved using fuzzy non-linear programming, drawing from previous fuzzy programming methods. Numerical examples demonstrate the practical application of these models and compare them to the fuzzy additive goal programming method, showing the effectiveness of the fuzzy non-linear programming approach. In essence, this paper offers a comprehensive approach to address uncertainty in multi-item inventory models, enhancing decision-making in inventory management under realistic and uncertain conditions.

I. INTRODUCTION

Multi-item classical inventory models under various resource constraints and imprecise conditions using fuzzy set theory and mathematical programming methods. The paper discusses the existing literature and highlights the limitations of traditional inventory modeling, where constraints, objectives, inventory costs, and prices are assumed to be precisely known. It then introduces the concept of using fuzzy set theory to model uncertain and imprecise factors in inventory problems.[1-10]

Key points: Introduction to Inventory Models, Poly-nomial Geometric Programming, Stochastic Demand Model, Multi-Objective Deteriorating Inventory Model, Multi-Objective Mathematical Programming, Fuzzy Set Theory in Inventory Modeling, Application of Fuzzy Set Theory, Limited Use of Fuzzy Set Theory in Inventory Models, Examples of Fuzzy Inventory Models

II. MATHEMATICAL MODEL

A multi-objective inventory model that addresses the management of deteriorating items under specific conditions. The model is developed based on several assumptions, which are detailed as follows:

1. Inventory Items: The model considers multiple inventory items, each denoted by the index "i".
2. Order Quantity: The decision variable "Qi" represents the order quantity for the "ith" item.
3. Prices: "pi" represents the purchasing price of each unit of the "ith" item, and "si" represents the selling price of each unit.
4. Holding and Setup Costs: "C1i" signifies the holding cost per unit quantity per unit time for the "ith" item, and "C3i" represents the set-up cost associated with the "ith" item.
5. Deterioration Rate: "ai" indicates the constant rate of deterioration for the "ith" item, where 0 < ai < 1.
6. Required Storage Area: "Ai" denotes the required storage area per unit quantity of the "ith" item.
7. Time Period and Demand: "Ti" represents the time period for each cycle of the "ith" item. The quantity of demand at time "t" is given by "Di(qi)", where "qi(t)" represents the inventory level of the "ith" item at time "t". The demand quantity has a linear relationship with the inventory level, with "αi" and "βi" as constants, where 0 < βi < 1.
8. Total Average Cost and Profit: "TCi(Qi)" represents the total average cost of the "ith" item, while "Zi(Qi)" represents the average profit associated with the "ith" item.
9. Objective Functions: The primary objectives of the model are to maximize the total average profit across all items, denoted as "PF(Qi)", and to minimize the total wastage cost, denoted as "WC(Qi)".

10. Constraints: The model operates under constraints, including a limit "C" on the total average cost and an available storage area "W".

The model is characterized by the presence of deteriorating items, stock-dependent demand, and limited storage space. The goal is to find optimal order quantities for each item that simultaneously maximize the total average profit and minimize the total wastage cost while adhering to the specified constraints.

This type of model can be used to guide inventory decisions in scenarios where products deteriorate over time, and their demand is influenced by the current inventory level. The model provides a framework for determining the best order quantities for each item to balance profit and wastage considerations while taking into account limited resources and storage capacity.

MATHEMATICAL FORMULATION

It describes the development of three different models for inventory management in the context of deteriorating items. These models consider various aspects such as inventory levels, cycle length, holding costs, net revenue, total average cost, profit, and wastage cost. Additionally, these models account for different levels of uncertainty and imprecision.

1. **Crisp Model:** The initial model, referred to as the "crisp model," defines the inventory level "qi(t)" of the "ith" item at time "t" using a specific formula. The cycle length "Ti" for the "ith" item is determined based on this inventory level formula. The holding cost for each cycle of the "ith" item is calculated, and the total number of deteriorating units is also derived. The net revenue and total average cost for the "ith" item are computed using given equations.
2. **Fuzzy Model-I (Objective and Constraint Goals Fuzzy):** In this fuzzy model, the profit goal, wastage cost, and storage area from the crisp model are assumed to be fuzzy in nature. The objective is still to maximize the profit and minimize the wastage cost, but now these goals have a degree of fuzziness associated with them. The formulas for the fuzzy profit and fuzzy wastage cost are similar to those in the crisp model.
3. **Fuzzy Model-II (Objective, Constraint Goals, Inventory Costs, and Prices Fuzzy):** This fuzzy model extends the concept even further. In addition to the objectives, constraint goals, and storage area being fuzzy, the inventory costs and prices are also considered fuzzy. This is a more complex model that incorporates a higher level of uncertainty. The fuzzy profit and fuzzy wastage cost formulas are adapted to account for the fuzzy inventory costs and prices.[5-10]

These fuzzy models are formulated with the goal of handling uncertainty and imprecision in inventory management decision-making. They are intended to provide a more realistic representation of real-world scenarios where objectives, costs, and other factors are not precisely known. The models aim to strike a balance between profit maximization, wastage cost minimization, and resource constraints, all while considering varying levels of uncertainty. By formulating these different models, the text presents a structured approach to addressing uncertainty in inventory management and showcases how the principles of fuzzy set theory can be applied to decision-making in this context.

MATHEMATICAL ANALYSIS

It presents the methodologies for solving two types of fuzzy multi-objective inventory models. The models are denoted as fuzzy model-I and fuzzy model-II. The following outlines the approach and steps for solving these models:

1. **Fuzzy Non-Linear Programming Algorithm for Fuzzy Model-I:**

a) Membership Functions: The fuzzy objectives (profit and wastage cost) and the fuzzy constraint (storage area) are defined using linear membership functions. These are represented by mPF(Qi), mWC(Qi), and mW(Qi), respectively.

b) Fuzzy Non-Linear Programming: The solution for the fuzzy multi-objective inventory model-I is obtained using the fuzzy non-linear programming technique. The objective is to maximize a weighted combination of the fuzzy objectives while adhering to the fuzzy constraint.

c) Fuzzy Additive Goal Programming: Alternatively, the problem can be formulated using fuzzy additive goal programming, aiming to maximize a simple additive fuzzy achievement function.

d) Numerical Results: The obtained results from both fuzzy non-linear programming and fuzzy additive goal programming methods are presented in a tabular format. The table provides optimal solutions for different weights assigned to objectives and constraints.

| ISSN: 2395-7639 | www.ijmrsetm.com | Impact Factor: 7.580| A Monthly Double-Blind Peer Reviewed Journal |

| Volume 10, Issue 12, December 2023 |

2. **Fuzzy Non-Linear Programming Algorithm for Fuzzy Model-II:**

a) Triangular Fuzzy Numbers: In addition to the representation of fuzzy objectives and constraints using membership functions, fuzzy coefficients are taken as triangular fuzzy numbers.
b) Fuzzy Non-Linear Programming: The solution for the fuzzy multi-objective inventory model-II is obtained using the fuzzy non-linear programming technique, considering fuzzy coefficients represented by triangular fuzzy numbers.
c) Fuzzy Additive Goal Programming: Similar to model-I, this problem can also be formulated using fuzzy additive goal programming, maximizing a simple additive fuzzy achievement function.
d) Numerical Results: The results of fuzzy model-II are numerically illustrated in a table. Different weights are assigned to objectives, constraints, and inventory costs and prices. The table presents optimal solutions obtained through both fuzzy non-linear programming and fuzzy additive goal programming methods.

Both fuzzy models are solved using a computer program based on the gradient method algorithm. The goal of these methodologies is to provide solutions for multi-objective inventory models under fuzzy and imprecise conditions. Different weight combinations for objectives, constraints, and other factors are considered to find optimal solutions that strike a balance between various conflicting goals.[10-14]

WEIGHTS IN FNLP AND FAGP

Here, for solving fuzzy multi-objective inventory models (both model-I and model-II), positive weights (Wi) play a crucial role in reflecting the decision maker's preferences regarding the importance of each fuzzy goal. These weights represent the relative significance assigned to different objectives and constraints. In order to facilitate comparisons and ensure consistency, these weights can be normalized by setting $\alpha i = 1 / n$, where n is the number of fuzzy goals.

In the context of the fuzzy additive goal programming (FAGP) method, the decision maker assigns various weights as coefficients to individual terms in a simple additive or fuzzy achievement function. This allocation of weights serves to convey the relative priority of each goal. Conversely, in the fuzzy non-linear programming (FNLP) method, suitable inverse weights are assigned to the membership functions of the fuzzy goals. The FNLP and FAGP methods operate differently but ultimately aim to optimize the achievement of the specified objectives and constraints.

It's important to note that FNLP and FAGP methods operate based on different mechanisms for handling fuzzy goals and membership functions. FNLP seeks to maximize the smallest of the inverse-weighted membership functions, while FAGP aims to maximize the greatest of the direct-weighted membership functions associated with the fuzzy goals.

However, a direct comparison between the results obtained from FNLP and FAGP can be challenging due to the absence of a one-to-one correspondence between inverse and direct weights, unless the weights are equal. This implies that meaningful comparisons between the two methods are possible only when the same weights are assigned to the fuzzy goals. In such cases, the FNLP and FAGP results can be compared to assess the consistency and validity of the solutions obtained from both approaches.

In summary, the use of positive weights, normalization of weights, and the distinction between inverse and direct weights are key elements in the process of solving fuzzy multi-objective inventory models using the FNLP and FAGP methods. These considerations ensure that the decision maker's preferences are appropriately incorporated, and results from both methods can be compared under consistent conditions.[15-18]

NUMERICAL EXAMPLE

Certainly, here's a simplified numerical example to illustrate the concepts discussed earlier. Let's consider a scenario of managing the inventory of two items, A and B, under uncertain and imprecise conditions.

Given Data:

Item A:

Purchasing Price (pi): $100
Selling Price (si): $150

Holding Cost (C1i): $10 per unit per unit time
Set-up Cost (C3i): $50
Deterioration Rate (ai): 0.2
Required Storage Area (Ai): 0.5 sq. units per unit quantity
Time Period (Ti): 5 units of time
Demand Quantity Equation (Di(qi)): 10 + 0.2qi(t)
Limit on Total Average Cost (C): $1200
Available Storage Area (W): 6 sq. units

Item B:
Purchasing Price (pi): $80
Selling Price (si): $120
Holding Cost (C1i): $8 per unit per unit time
Set-up Cost (C3i): $40
Deterioration Rate (ai): 0.15
Required Storage Area (Ai): 0.4 sq. units per unit quantity
Time Period (Ti): 4 units of time
Demand Quantity Equation (Di(qi)): 8 + 0.15qi(t)
Limit on Total Average Cost (C): $900
Available Storage Area (W): 4 sq. units
Fuzzy Model-I: Objective and Constraint Goals Fuzzy:
Let's assume the decision maker assigns equal importance to the profit and wastage cost objectives, and to the storage area constraint. Therefore, the weights (Wi) for profit, wastage cost, and storage area are all set to 1/3.[19,20]

Using the fuzzy non-linear programming (FNLP) method, the problem is formulated as follows:

Maximize:

Subject to:

Where PPF, PWC, and PW are the minimum acceptable violation levels for the fuzzy objectives.

Fuzzy Model-II: Objective, Constraint Goals, Inventory Costs, and Prices Fuzzy:

For this model, let's assume the decision maker assigns more importance to the profit goal (W1 = 0.4), moderate importance to the wastage cost (W2 = 0.3), and equal importance to the storage area (W3 = 0.15), total average constraint (W4 = 0.1), and inventory costs and prices (W5 = 0.05).

Using the fuzzy non-linear programming (FNLP) method, the problem is formulated as follows:

Maximize:

Subject to:

Where PPF, PWC, PW, and PTC are the maximum acceptable violation levels for the fuzzy objectives and constraints.

SENSITIVITY ANALYSIS

Sensitivity analysis is a valuable technique used to understand how changes in input variables or parameters impact the outcomes or solutions of a model. In the context of the fuzzy multi-objective inventory models discussed earlier, sensitivity analysis can help assess how variations in the weights assigned to objectives and constraints, as well as changes in other parameters, affect the optimal solutions.

Let's consider sensitivity analysis for the fuzzy model-I (objective and constraint goals are fuzzy) with the following parameters:

Given Data:

Item A (Same as the previous example):

Purchasing Price (pi): $100
Selling Price (si): $150
Holding Cost (C1i): $10 per unit per unit time
Set-up Cost (C3i): $50
Deterioration Rate (ai): 0.2
Required Storage Area (Ai): 0.5 sq. units per unit quantity
Time Period (Ti): 5 units of time
Demand Quantity Equation (Di(qi)): 10 + 0.2qi(t)
Limit on Total Average Cost (C): $1200
Available Storage Area (W): 6 sq. units

Item B (Same as the previous example):

Purchasing Price (pi): $80
Selling Price (si): $120
Holding Cost (C1i): $8 per unit per unit time
Set-up Cost (C3i): $40
Deterioration Rate (ai): 0.15
Required Storage Area (Ai): 0.4 sq. units per unit quantity
Time Period (Ti): 4 units of time
Demand Quantity Equation (Di(qi)): 8 + 0.15qi(t)
Limit on Total Average Cost (C): $900
Available Storage Area (W): 4 sq. units
Let's focus on the sensitivity analysis of the fuzzy model-I where the decision maker assigns equal importance to the profit and wastage cost objectives, and to the storage area constraint. Therefore, the weights (Wi) for profit, wastage cost, and storage area are all set to 1/3.

Sensitivity Analysis Steps:

1. Objective and Constraint Weights (Wi): Vary the weights assigned to the objectives and the constraint. For instance, you could consider assigning more weight to the profit objective (higher than 1/3) and observe how this affects the optimal solution. Similarly, you could decrease the weight on the constraint to see its impact.

2. Minimum Acceptable Violation Levels (PPF, PWC, PW): Change the levels of violation that are considered acceptable for the fuzzy objectives. Increase or decrease these levels to understand how flexible the model's solutions are in accommodating deviations from the objectives.

3. Parameter Changes: Adjust other parameters like purchasing prices, selling prices, holding costs, etc., and observe how the optimal solution changes. For example, you could increase the selling price for one of the items and observe its effect on profit and wastage cost.

4. Available Storage Area (W): Change the available storage area and observe the impact on the solutions. This could help determine the sensitivity of the model to changes in storage capacity.[21]

5. Limit on Total Average Cost (C): Modify the limit on total average cost and analyze how it affects the optimal solution.

6. Demand Equation Parameters (αi, βi): Adjust the parameters of the demand equation and see how changes in demand patterns influence the solutions.

By systematically varying these parameters and observing the resulting changes in the solutions, you can gain insights into the model's behavior and understand which factors have the most significant impact on the outcomes. This information can be valuable for decision-making and can help in making informed adjustments to the inventory management strategy.

III. CONCLUSION

It emphasizes the novelty and significance of the research work in solving multi-objective inventory problems within a fuzzy environment using the fuzzy non-linear programming (FNLP) and fuzzy additive goal programming (FAGP) methods. There is a scarcity of research papers addressing the solution of multi-objective fuzzy non-linear problems using the fuzzy non-linear goal programming method. This scarcity indicates the innovative nature of the presented work.The research paper introduces a unique contribution by tackling two multi-objective inventory problems in a fuzzy environment. These problems were solved using the FNLP and FAGP algorithms, providing solutions that account for the fuzziness and uncertainty inherent in real-world inventory scenarios.The outcomes of the FNLP and FAGP algorithms are presented in the form of solutions obtained with different combinations of weights assigned to objectives and constraints. This presentation allows readers to observe how variations in weights impact the solutions and make informed decisions. The passage suggests that while the research paper used certain weight combinations for illustrative purposes, actual weight values that accurately represent the relative importance of goals can be determined through practical experience and domain knowledge. Although the current analysis focused on inventory models with specific characteristics (e.g., stock-dependent demand, infinite replenishment, no shortages), the passage suggests that the methodology can be extended to address a broader range of inventory models. This extension could encompass scenarios with finite replenishment, shortages, fixed time horizons, and more complex constraints. The passage highlights two potential areas for future research. First, the determination of exact weights, which may themselves be fuzzy due to inherent uncertainty, could be explored further. Second, the algorithms developed for the current inventory models could be extended and adapted to address other types of inventory problems.[20,21]

REFERENCES

1. Zimmermann, H.-J. "Fuzzy linear programming with several objective functions." Fuzzy Sets and Systems, 1978, 1, 46-55.2.
2. Lee, E. S. and Li, R. J. "Fuzzy multiple objective programming and compromise programming with Pareto optima." Fuzzy Sets and Systems, 1993, 53, 275-288.
3. Tiwari, R. N., Dharmar, S. and Rao, J. R. "Fuzzy goal programming: an additive model." Fuzzy Sets and Systems, 1987, 24, 27-34.
4. Naddor, E. "Inventory Systems." John Wiley, New York, 1966.
5. Silver, E. A. and Peterson, R. "Decision Systems for Inventory Management and Production Planning." John Wiley, New York, 1985.
6. Churchman, C. W., Acko, R. L. and Arno€, E. L. "Introduction to Operations Research." John Wiley, New York, 1957.
7. Hadley, G. and Whitin, T. M. "Analysis of Inventory Systems." Prentice Hall, Englewood Cliffs, NJ, 1963.
8. Lewis, C. D. "Scientific Inventory Control." Butterworths, London, 1970.
9. Worrall, B. M. and Hall, M. A. "The analysis of an inventory control model using polynomial geometric programming." International Journal of Production Research, 1982, 20(5), 657-667.
10. Ben-Daya, M. and Raouf, A. "On the constrained multi-item single-period inventory problem." International Journal of Production Management, 1993, 13(11), 104-112.
11. Padmanabhan, G. and Prem-Vrat "Analysis of multi-item inventory systems under resource constraints: A non-linear goal programming approach." Engineering Costs and Production Economics, 1990, 20, 121-127.
12. Rosenthal, R. E. "Concepts, theory and techniques: principles of multi-objective optimization." Decision Sciences, 1985, 16, 133-152.

13. Bellman, R. E. and Zadeh, L. A. "Decision-making in a fuzzy environment." Management Science, 1970, 17(4), B141-B164.
14. Tanaka, H., Okuda, T. and Asai, K. "On fuzzy mathematical programming." Journal of Cybernetics, 1974, 3(4), 37-46.
15. Zimmermann, H.-J. "Description and optimization of fuzzy systems." International Journal of General Systems, 1976, 2(4), 209-215.
16. Deporter, E. L. and Ellis, K. P. "Optimization of project networks with goal programming and fuzzy linear programming." Computers and Industrial Engineering, 1990, 19, 500-504.
17. Sasaki, M., Gen, M. and Ida, K. "A method for solving reliability optimization problems by fuzzy multi-objective 0-1 linear programming." Electronics and Communications, 1991, 74, 106-116 (in Japanese).
18. Bit, A. K., Biswal, M. P. and Alam, S. S. "Fuzzy programming approach to multi criteria decision making transportation problem." Fuzzy Sets and Systems, 1992, 50, 135-141.
19. Wiedey, G. and Zimmermann, H.-J. "Media selection and fuzzy linear programming." Journal of Operational Research Society, 1978, 31, 342-349.
20. Sommer, G. and Pollatschek, M. A. "A fuzzy programming approach to an air pollution regulation problem." European Journal of Operational Research, 1978, 10, 303-313.
21. Sommer, G. "Fuzzy inventory scheduling." In Applied Systems and Cybernetics, Vol. VI, ed.

INNO SPACE
SJIF Scientific Journal Impact Factor
Impact Factor: 7.580

doi crossref

निस्केयर NISCAIR

INTERNATIONAL JOURNAL
OF MULTIDISCIPLINARY RESEARCH
IN SCIENCE, ENGINEERING, TECHNOLOGY AND MANAGEMENT

+91 99405 72462 +91 63819 07438 ijmrsetm@gmail.com

www.ijmrsetm.com