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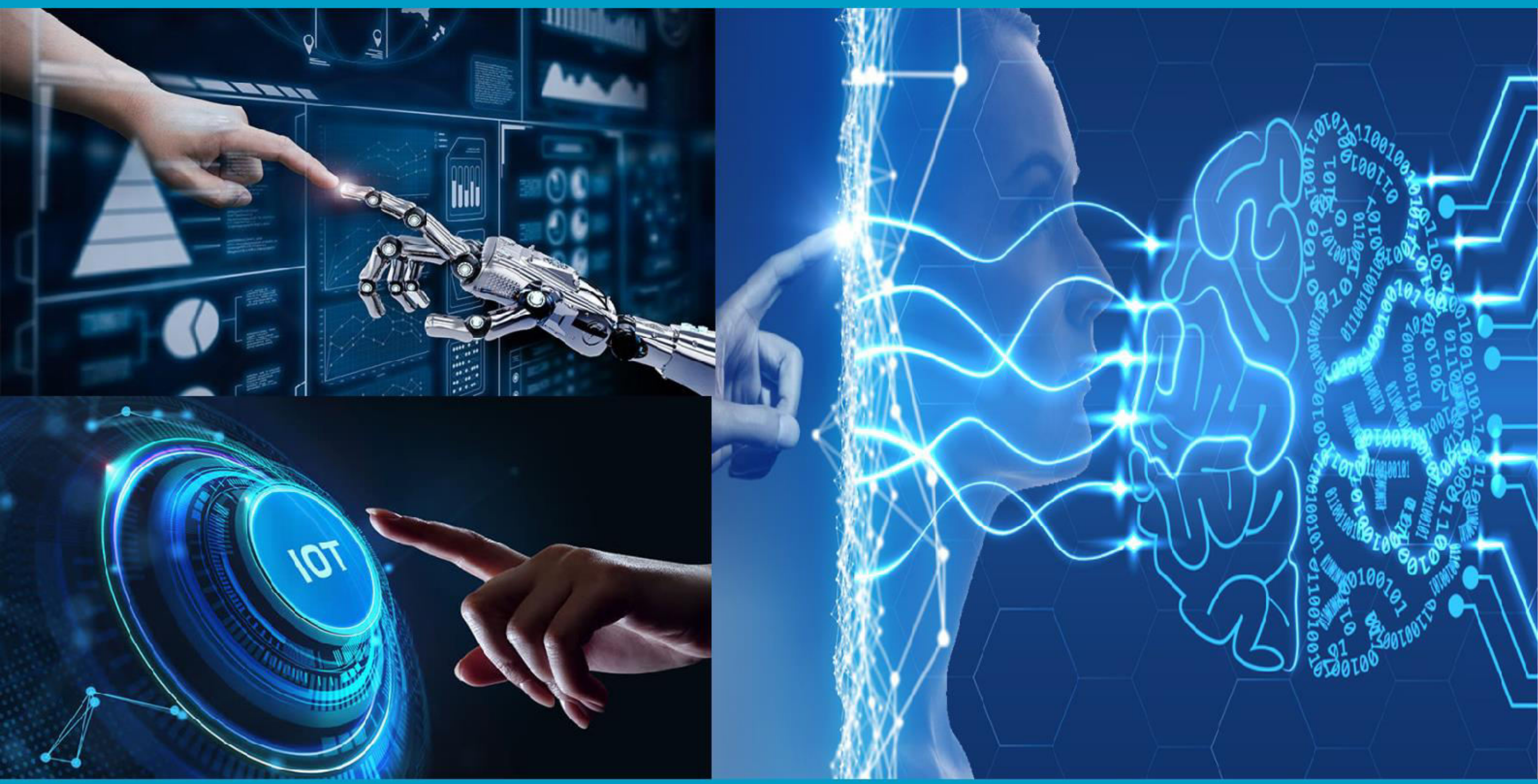

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Euclidian Geometry in the Context of Propagation of Plane Geometry in Analyzing Some Basic Geometrical Shapes

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ABSTRACT: Euclidean geometry is the study of geometrical shapes (plane and solid) and figures based on different axioms and theorems. It is basically introduced for flat surfaces or plane surfaces. Euclidean geometry is the study of plane and solid figures on the basis of axioms and theorems employed by the Greek mathematician Euclid (c. 300 BCE). Indeed, until the second half of the 19th century, when non-Euclidean geometries attracted the attention of mathematicians, Geometry meant Euclidean geometry. It is the most typical expression of general mathematical thinking. Rather than the memorization of simple algorithms to solve equations by repetition, it demands true insight into the subject, clever ideas for applying theorems in special situations, an ability to generalize from known facts, and an insistence on the importance of proof. The modern version of Euclidean geometry is the theory of Euclidean (coordinate) spaces of multiple dimensions, where distance is measured by a suitable generalization of the Pythagorean theorem.

KEYWORDS: Geometry, Euclidian, flat, space, plane, surface.

I. INTRODUCTION

1.1 Fundamentals of Euclidian Geometry

Euclidean geometry is the study of flat shapes or figures of flat surfaces and straight lines in two dimensions.Euclid realized that a rigorous development of Geometry must start with the foundations. Hence, he began the elements with some undefined terms, such as a point is that which has no part and a line is a length without breadth. Proceeding from these terms, he defined further ideas such as angles, circles, triangles, and various other polygons and figures. For example, an angle was defined as the inclination of two straight lines, and a circle was a plane figure consisting of all points that have a fixed distance (radius) from a given centre.

As a basis for further logical deductions, Euclid proposed five common notions, such as “things equal to the same thing are equal,” and five unprovable but intuitive principles known variously as postulates or axioms. Stated in modern terms, the axioms are as follows:

- 1. Given two points, there is a straight line that joins them.
- 2. A straight line segment can be prolonged indefinitely.
- 3. A circle can be constructed when a point for its centre and a distance for its radius are given.
- 4. All right angles are equal.
- 5. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, will meet on that side on which the angles are less than the two right angles.

The fifth axiom became known as the “parallel postulate,” since it provided a basis for the uniqueness of parallel lines. (It also attracted great interest because it seemed less intuitive or self-evident than the others. In the 19th century, Carl Friedrich Gauss, János Bolyai, and Nikolay Lobachevsky all began to experiment with this postulate, eventually arriving at new, non-Euclidean, geometries.) All five axioms provided the basis for numerous provable statements, or theorems, on which Euclid built his geometry.

Euclidean geometry is better explained especially for the shapes of geometrical figures and planes. This part of geometry was employed by the Greek mathematician Euclid, who has also described it in his book, **Elements**. Therefore, this geometry is also called Euclid geometry.

The axioms or postulates are the assumptions that are obvious universal truths, they are not proved. Euclid has introduced the geometry fundamentals like geometric shapes and figures in his book elements and has stated 5 main

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Axioms or Postulates. Here, in this paper, discussion is done on the definition of Euclidean geometry, its elements, axioms and five important postulates.
Euclidean Geometry is considered an axiomatic system, where all the theorems are derived from a small number of simple axioms. Since the term Geometry deals with things like points, lines, angles, squares, triangles, and other shapes, Euclidean Geometry is also known as plane geometry. It deals with the properties and relationships between all things.

Plane Geometry

1. Congruence of triangles
2. Similarity of triangles
3. Areas
4. Pythagorean theorem
5. Circles
6. Regular polygons
7. Conic sections

Solid Geometry

1. Volume
2. Regular solids

Examples of Euclidean Geometry

The two common examples of Euclidean geometry are angles and circles. Angles are said as the inclination of two straight lines. A circle is a plane figure, that has all the points at a constant distance (called the radius) from the center.

Euclidean and Non-Euclidean Geometry

There is a difference between Euclidean and non-Euclidean geometry in the nature of parallel lines. In Euclidean geometry, for the given point and line, there is exactly a single line that passes through the given points in the same plane and it never intersects.
Non-Euclidean is different from Euclidean geometry. The spherical geometry is an example of non-Euclidean geometry because lines are not straight here.

Properties of Euclidean Geometry

- It is the study of plane geometry and solid geometry
- It defined point, line and a plane
- A solid has shape, size, position, and can be moved from one place to another.
- The interior angles of a triangle add up to 180 degrees
- Two parallel lines never cross each other
- The shortest distance between two points is always a straight line

II. REVIEW OF RELATED LITERATURE

2.1 History of Euclid Geometry

The excavations at Harappa and Mohenjo-Daro depict the extremely well-planned towns of Indus Valley Civilization (about 3300-1300 BC). The flawless construction of Pyramids by the Egyptians is yet another example of extensive use of geometrical techniques used by the people back then. In India, the Sulba Sutras, textbooks on Geometry depict that the Indian Vedic Period had a tradition of Geometry. The development of geometry was taking place gradually, when Euclid, a teacher of mathematics, at Alexandria in Egypt, collected most of these evolutions in geometry and compiled it into his famous treatise, which he named Elements.
The uniqueness of Euclidean geometry, and the absolute identification of Mathematics with reality, was broken in the 19th century when Nikolay Lobachevsky and János Bolyai (1802–60) independently discovered that altering the parallel postulate resulted in perfectly consistent Non-Euclidean Geometries.

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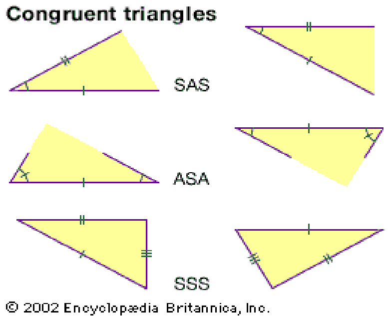
Plane geometry

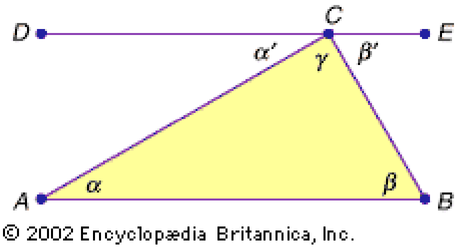
Congruence of triangles



congruent triangles

Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion, and the congruence theorems specify the conditions under which this can occur. The first such theorem is the side-angle-side (SAS) theorem: If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, the triangles are congruent. Following this, there are corresponding angle-side-angle (ASA) and side-side-side (SSS) theorems.

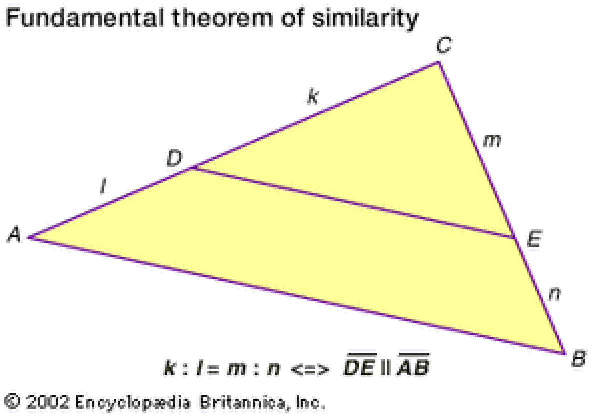
The first very useful theorem derived from the axioms is the basic symmetry property of isosceles triangles—i.e., that two sides of a triangle are equal if and only if the angles opposite them are equal. Euclid's proof of this theorem was once called Pons Asinorum (Bridge of Asses), supposedly because mediocre students could not proceed across it to the farther reaches of geometry. The Bridge of Asses opens the way to various theorems on the congruence of triangles.



Proof that the sum of the angles in a triangle is 180 degrees

The parallel postulate is fundamental for the proof of the theorem that the sum of the angles of a triangle is always 180 degrees. A simple proof of this theorem was attributed to the Pythagoreans.

Similarity of triangles



Fundamental theorem of similarity

As indicated above, congruent figures have the same shape and size. Similar figures, on the other hand, have the same shape but may differ in size. Shape is intimately related to the notion of proportion, as ancient Egyptian artisans observed long ago. Segments of lengths a, b, c, and d are said to be proportional if a:b = c:d (read, a is to b as c is to d;

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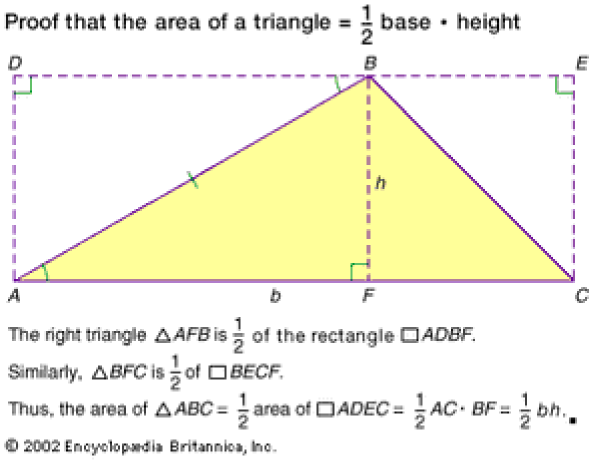
in older notation a:b::c:d). The fundamental theorem of similarity states that a line segment splits two sides of a triangle into proportional segments if and only if the segment is parallel to the triangle's third side.

The similarity theorem may be reformulated as the AAA (angle-angle-angle) similarity theorem: two triangles have their corresponding angles equal if and only if their corresponding sides are proportional. Two similar triangles are related by a scaling (or similarity) factor s: if the first triangle has sides a, b, and c, then the second one will have sides sa, sb, and sc. In addition to the universal use of scaling factors on construction plans and geographic maps, similarity is fundamental to trigonometry.

Areas



Area of a triangle

Just as a segment can be measured by comparing it with a unit segment, the area of a polygon or other plane figure can be measured by comparing it with a unit square. The common formulas for calculating areas reduce this kind of measurement to the measurement of certain suitable lengths. The simplest case is a rectangle with sides a and b, which has area ab. By putting a triangle into an appropriate rectangle, one can show that the area of the triangle is half the product of the length of one of its bases and its corresponding height—$bh/2$. One can then compute the area of a general polygon by dissecting it into triangular regions. If a triangle (or more general figure) has area A, a similar triangle (or figure) with a scaling factor of s will have an area of s^2A.

Pythagorean theorem

For a triangle $\triangle ABC$ the Pythagorean theorem has two parts: (1) if $\angle ACB$ is a right angle, then $a^2 + b^2 = c^2$; (2) if $a^2 + b^2 = c^2$, then $\angle ACB$ is a right angle. For an arbitrary triangle, the Pythagorean theorem is generalized to the law of cosines: $a^2 + b^2 = c^2 - 2ab \cos(\angle ACB)$. When $\angle ACB$ is 90 degrees, this reduces to the Pythagorean theorem because $\cos(90°) = 0$.

Since Euclid, a host of professional and amateur mathematicians (even U.S. President James Garfield) have found more than 300 distinct proofs of the Pythagorean theorem. Despite its antiquity, it remains one of the most important theorems in Mathematics. It enables one to calculate distances or, more important, to define distances in situations far more general than elementary geometry. For example, it has been generalized to multidimensional Vector Spaces.

Circles

A chord AB is a segment in the interior of a circle connecting two points (A and B) on the circumference. When a chord passes through the circle's centre, it is a diameter, d. The circumference of a circle is given by πd, or $2\pi r$ where r is the radius of the circle; the area of a circle is πr^2. In each case, π is the same constant (3.14159…). The Greek mathematician Archimedes (c. 287–212/211 bce) used the method of exhaustion to obtain upper and lower bounds for π by circumscribing and inscribing regular polygons about a circle.

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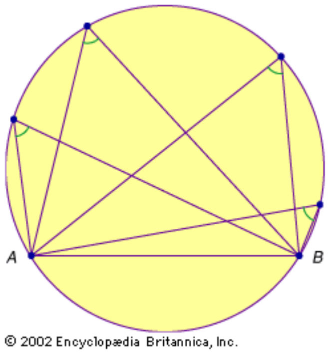

Chord in a circle

A semicircle has its end points on a diameter of a circle. Thales (flourished 6th century BCE) is generally credited with having proved that any angle inscribed in a semicircle is a right angle; that is, for any point C on the semicircle with diameter AB, $\angle ACB$ will always be 90 degrees (*see*Sidebar: Thales' Rectangle). Another important theorem states that for any chord AB in a circle, the angle subtended by any point on the same semiarc of the circle will be invariant. Slightly modified, this means that in a circle, equal chords determine equal angles, and vice versa.

Summarizing the above material, the five most important theorems of plane Euclidean geometry are: the sum of the angles in a triangle is 180 degrees, the Bridge of Asses, the fundamental theorem of similarity, the Pythagorean theorem, and the invariance of angles subtended by a chord in a circle. Most of the more advanced theorems of plane Euclidean geometry are proved with the help of these theorems.

Regular polygons

A polygon is called regular if it has equal sides and angles. Thus, a regular triangle is an equilateral triangle, and a regular quadrilateral is a square. A general problem since antiquity has been the problem of constructing a regular n-gon, for different n, with only ruler and compass. For example, Euclid constructed a regular pentagon by applying the above-mentioned five important theorems in an ingenious combination.

Conic Sections and Geometric Art

The most advanced part of plane Euclidean geometry is the theory of the conic sections (the ellipse, the parabola, and the hyperbola). Much as the Elements displaced all other introductions to geometry, the Conics of Apollonius of Parga (c. 240–190 BCE), known by his contemporaries as "the Great Geometer," was for many centuries the definitive treatise on the subject.

MedievalIslamic artists explored ways of using geometric figures for decoration. For example, the decorations of the Alhambra of Granada, Spain, demonstrate an understanding of all 17 of the different "Wallpaper groups" that can be used to tile the plane. In the 20th century, internationally renowned artists such as Josef Albers, Max Bill, and Sol LeWitt were inspired by motifs from Euclidean geometry.

Solid geometry

The most important difference between plane and solid Euclidean geometry is that human beings can look at the plane from above, whereas three-dimensional space cannot be looked at from outside. Consequently, intuitive insights are more difficult to obtain for solid geometry than for plane geometry.

Some concepts, such as proportions and angles, remain unchanged from plane to solid geometry. For other familiar concepts, there exist analogies—most noticeably, volume for area and three-dimensional shapes for two-dimensional shapes (sphere for circle, tetrahedron for triangle, box for rectangle). However, the theory of tetrahedra is not nearly as rich as it is for triangles. Active research in higher-dimensional Euclidean geometry includes convexity and sphere packings and their applications in cryptology and crystallography.

Volume

As explained above, in plane geometry the area of any polygon can be calculated by dissecting it into triangles. A similar procedure is not possible for solids. In 1901 the German mathematician Max Dehn showed that there exist a cube and a tetrahedron of equal volume that cannot be dissected and rearranged into each other. This means that calculus must be used to calculate volumes for even many simple solids such as pyramids.

Regular solids

Regular polyhedral are the solid analogies to regular polygons in the plane. Regular polygons are defined as having equal (congruent) sides and angles. In analogy, a solid is called regular if its faces are congruent regular polygons and

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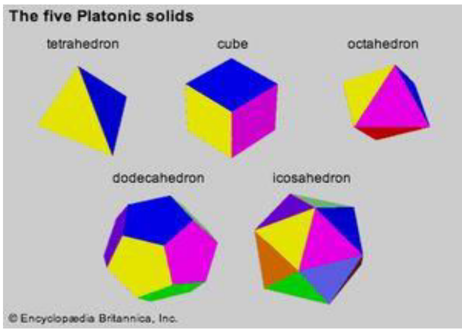
its polyhedral angles (angles at which the faces meet) are congruent. This concept has been generalized to higher-dimensional (coordinate)



Platonic solids

Whereas in the plane there exist (in theory) infinitely many regular polygons, in three-dimensional space there exist exactly five regular polyhedra. These are known as the Platonic solids: the tetrahedron, or pyramid, with 4 triangular faces; the cube, with 6 square faces; the octahedron, with 8 equilateral triangular faces; the dodecahedron, with 12 pentagonal faces; and the icosahedron, with 20 equilateral triangular faces.

In four-dimensional space there exist exactly six regular polytopes, five of them generalizations from three-dimensional space. In any space of more than four dimensions, there exist exactly three regular polytopes—the generalizations of the tetrahedron, the cube, and the octahedron.

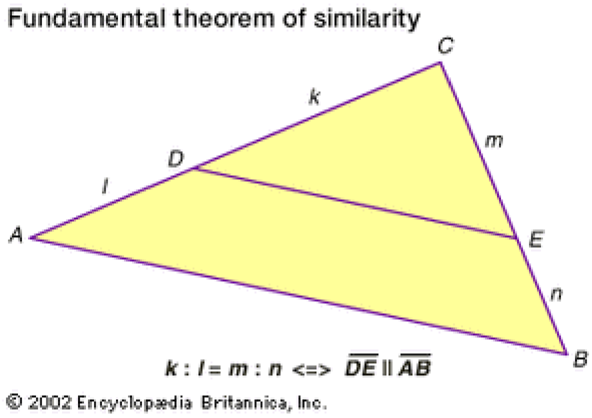
Parallel postulate

Parallel postulate, One of the five postulates, or axioms, of Euclid underpinning Euclidean geometry. It states that through any given point not on a line there passes exactly one line parallel to that line in the same plane. Unlike Euclid's other four postulates, it never seemed entirely self-evident, as attested by efforts to prove it through the centuries. The uniqueness of Euclidean geometry, and the absolute identification of mathematics with reality, was broken in the 19th century when Nikolay Lobachevsky and János Bolyai (1802–60) independently discovered that altering the parallel postulate resulted in perfectly consistent non-Euclidean geometries.

Euclidean space

In geometry, a two- or three-dimensional space in which the axioms and postulates of Euclidean geometry apply; also, a space in any finite number of dimensions, in which points are designated by coordinates (one for each dimension) and the distance between two points is given by a distance formula. The only conception of physical space for over 2,000 years, it remains the most compelling and useful way of modeling the world as it is experienced. Though non-Euclidean spaces, such as those that emerge from elliptic geometry and hyperbolic geometry, have led scientists to a better understanding of the universe and of mathematics itself, Euclidean space remains the point of departure for their study.

Parallel lines and the projection of infinity



fundamental theorem of similarity

A theorem from Euclid's Elements (c. 300 BC) states that if a line is drawn through a triangle such that it is parallel to one side (*see* the figure), then the line will divide the other two sides proportionately; that is, the ratio of segments on

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each side will be equal. This is known as the proportional segments theorem, or the fundamental theorem of similarity, and for triangle *ABC*, shown in the diagram, with line segment *DE* parallel to side *AB*, the theorem corresponds to the mathematical expression $CD/DA = CE/EB$.

Euclid's Five Postulates

Before discussing Postulates in Euclidean Geometry, let us discuss a few terms as listed by Euclid in his book 1 of the 'Elements'. The postulated statements of these are:

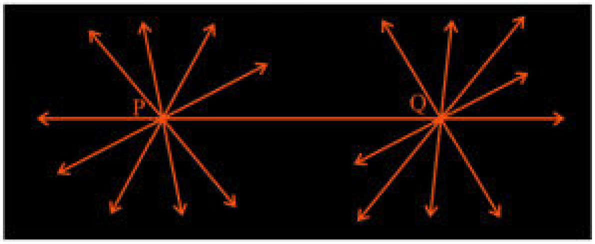
- Assume the three steps from solids to points as solids-surface-lines-points. In each step, one dimension is lost.
- A solid has 3 dimensions, the surface has 2, the line has 1 and the point is dimensionless.
- A point is anything that has no part, a breadthless length is a line and the ends of a line point.
- A surface is something that has length and breadth only.

It can be seen that the definition of a few terms needs extra specification. Now let us discuss these Postulates in detail.

Euclid's Postulate 1

"A straight line can be drawn from any one point to another point."

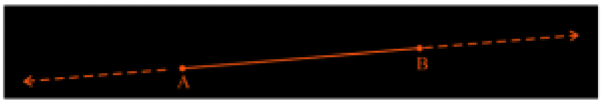
This postulate states that at least one straight line passes through two distinct points but he did not mention that there cannot be more than one such line. Although throughout his work he has assumed there exists only a unique line passing through two points.



Euclid's Postulate 2

"A terminated line can be further produced indefinitely."

In simple words what we call a line segment was defined as a terminated line by Euclid. Therefore, this postulate means that we can extend a terminated line or a line segment in either direction to form a line. In the figure given below, the line segment AB can be extended as shown to form a line.



Euclid's Postulate 3

"A circle can be drawn with any centre and any radius."

Any circle can be drawn from the end or start point of a circle and the diameter of the circle will be the length of the line segment.

Euclid's Postulate 4

"All right angles are equal to one another."

All the right angles (i.e., angles whose measure is 90°) are always congruent to each other i.e., they are equal irrespective of the length of the sides or their orientations.

Euclid's Postulate 5

"If a straight line falling on two other straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on the side on which the sum of angles is less than two right angles."

To learn More on 5th postulate, read: Euclid's 5th Postulate

Further, these Postulates and axioms were used by him to prove other geometrical concepts using deductive reasoning. No doubt the foundation of present-day geometry was laid by him and his book the 'Elements'.

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Difference between Euclidean and non-Euclidean Geometry

Euclidean geometry deals with figures of flat surfaces but all other figures which do not fall under this category come under non-Euclidean geometry. For example, a curved shape or spherical shape is a part of non-Euclidean geometry.

Three types of geometry

In the two-dimensional plane, there are majorly three types of geometries.
Euclidean (for flat surfaces)
Spherical (for curved surfaces)
Hyperbolic

Utilization of Euclidean Geometry

Euclidean geometry is majorly used in the field of architecture to build a variety of structures and buildings. Designing is the huge application of this geometry. Also, in surveying, it is used to do the levelling of the ground.

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