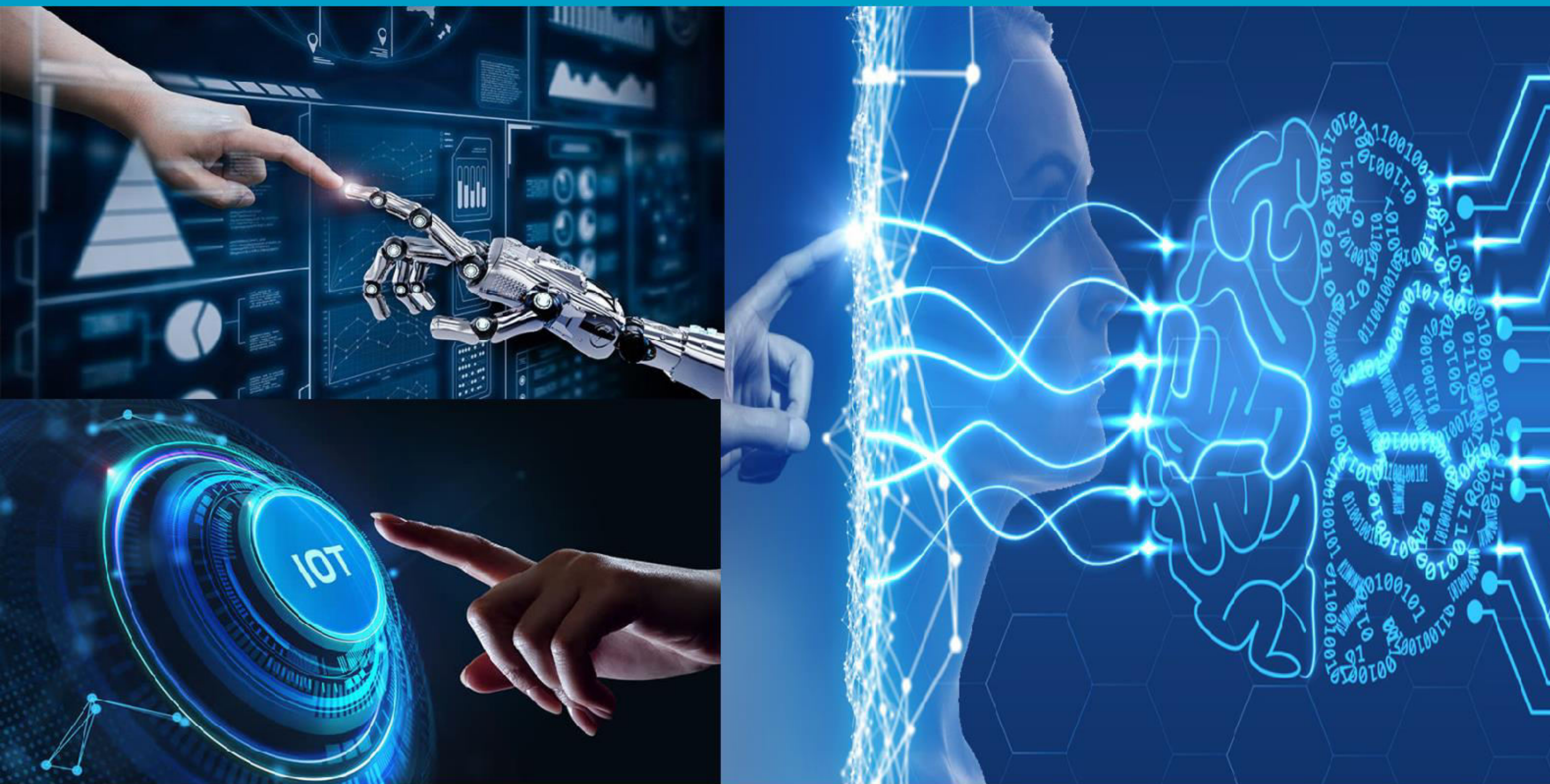
IJMRSETM

e-ISSN: 2395 - 7639



INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY RESEARCH

IN SCIENCE, ENGINEERING, TECHNOLOGY AND MANAGEMENT

Volume 8, Issue 12, December 2021

INTERNATIONAL STANDARD SERIAL NUMBER INDIA

Impact Factor: 7.580

International Journal of Multidisciplinary Research in Science, Engineering, Technology & Management (IJMRSETM)

| ISSN: 2395-7639 | www.ijmrsetm.com | Impact Factor: 7.580| A Monthly Double-Blind Peer Reviewed Journal |

| Volume 8, Issue 12, December 2021 |

| DOI: 10.15680/IJMRSETM.2021.0812009 |

Group Theory-A Review

Meetha Lal Meena

Assistant Professor, Mathematics, SCRS Govt. College, Sawai Madhopur, Rajasthan, India

ABSTRACT: In abstract algebra, **group theory** studies the algebraic structures known as groups. The concept of a group is central to abstract algebra: other well-known algebraic structures, such as rings, fields, and vector spaces, can all be seen as groups endowed with additional operations and axioms. Groups recur throughout mathematics, and the methods of group theory have influenced many parts of algebra. Linear algebraic groups and Lie groups are two branches of group theory that have experienced advances and have become subject areas in their own right.
Various physical systems, such as crystals and the hydrogen atom, and three of the four known fundamental forces in the universe, may be modelled by symmetry groups. Thus group theory and the closely related representation theory have many important applications in physics, chemistry, and materials science. Group theory is also central to public key cryptography.
The early history of group theory dates from the 19th century. One of the most important mathematical achievements of the 20th century[1] was the collaborative effort, taking up more than 10,000 journal pages and mostly published between 1960 and 2004, that culminated in a complete classification of finite simple groups.

KEYWORDS: algebra, group theory, cryptography, classification, lie groups, science, operations

I.INTRODUCTION

Group theory has three main historical sources: number theory, the theory of algebraic equations, and geometry. The number-theoretic strand was begun by Leonhard Euler, and developed by Gauss's work on modular arithmetic and additive and multiplicative groups related to quadratic fields. Early results about permutation groups were obtained by Lagrange, Ruffini, and Abel in their quest for general solutions of polynomial equations of high degree. Évariste Galois coined the term "group" and established a connection, now known as Galois theory, between the nascent theory of groups and field theory. In geometry, groups first became important in projective geometry and, later, non-Euclidean geometry. Felix Klein's Erlangen program proclaimed group theory to be the organizing principle of geometry.[1]

Galois, in the 1830s, was the first to employ groups to determine the solvability of polynomial equations. Arthur Cayley and Augustin Louis Cauchy pushed these investigations further by creating the theory of permutation groups. The second historical source for groups stems from geometrical situations. In an attempt to come to grips with possible geometries (such as euclidean, hyperbolic or projective geometry) using group theory, Felix Klein initiated the Erlangen programme. Sophus Lie, in 1884, started using groups (now called Lie groups) attached to analytic problems. Thirdly, groups were, at first implicitly and later explicitly, used in algebraic number theory.[2]
The different scope of these early sources resulted in different notions of groups. The theory of groups was unified starting around 1880. Since then, the impact of group theory has been ever growing, giving rise to the birth of abstract algebra in the early 20th century, representation theory, and many more influential spin-off domains. The classification of finite simple groups is a vast body of work from the mid 20th century, classifying all the finite simple groups.
The range of groups being considered has gradually expanded from finite permutation groups and special examples of matrix groups to abstract groups that may be specified through a presentation by generators and relations.[3]

II.PERMUTATION GROUPS

The first class of groups to undergo a systematic study was permutation groups. Given any set X and a collection G of bijections of X into itself (known as *permutations*) that is closed under compositions and inverses, G is a group acting on X. If X consists of n elements and G consists of *all* permutations, G is the symmetric group S_n; in general, any permutation group G is a subgroup of the symmetric group of X. An early construction due to Cayley exhibited any group as a permutation group, acting on itself ($X = G$) by means of the left regular representation.

In many cases, the structure of a permutation group can be studied using the properties of its action on the corresponding set. For example, in this way one proves that for $n \geq 5$, the alternating group A_n is simple, i.e. does not

| ISSN: 2395-7639 | www.ijmrsetm.com | Impact Factor: 7.580| A Monthly Double-Blind Peer Reviewed Journal |

| Volume 8, Issue 12, December 2021 |

| DOI: 10.15680/IJMRSETM.2021.0812009 |

admit any proper normal subgroups. This fact plays a key role in the impossibility of solving a general algebraic equation of degree $n \geq 5$ in radicals.[4]

Matrix groups

The next important class of groups is given by *matrix groups*, or linear groups. Here G is a set consisting of invertible matrices of given order n over a field K that is closed under the products and inverses. Such a group acts on the n-dimensional vector space K^n by linear transformations. This action makes matrix groups conceptually similar to permutation groups, and the geometry of the action may be usefully exploited to establish properties of the group G.[5]

Transformation groups

Permutation groups and matrix groups are special cases of transformation groups: groups that act on a certain space X preserving its inherent structure. In the case of permutation groups, X is a set; for matrix groups, X is a vector space. The concept of a transformation group is closely related with the concept of a symmetry group: transformation groups frequently consist of *all* transformations that preserve a certain structure.

The theory of transformation groups forms a bridge connecting group theory with differential geometry. A long line of research, originating with Lie and Klein, considers group actions on manifolds by homeomorphisms or diffeomorphisms. The groups themselves may be discrete or continuous.[6]

Abstract groups

Most groups considered in the first stage of the development of group theory were "concrete", having been realized through numbers, permutations, or matrices. It was not until the late nineteenth century that the idea of an abstract group as a set with operations satisfying a certain system of axioms began to take hold. A typical way of specifying an abstract group is through a presentation by *generators and relations*,

A significant source of abstract groups is given by the construction of a *factor group*, or quotient group, G/H, of a group G by a normal subgroup H. Class groups of algebraic number fields were among the earliest examples of factor groups, of much interest in number theory. If a group G is a permutation group on a set X, the factor group G/H is no longer acting on X; but the idea of an abstract group permits one not to worry about this discrepancy.[7]

The change of perspective from concrete to abstract groups makes it natural to consider properties of groups that are independent of a particular realization, or in modern language, invariant under isomorphism, as well as the classes of group with a given such property: finite groups, periodic groups, simple groups, solvable groups, and so on. Rather than exploring properties of an individual group, one seeks to establish results that apply to a whole class of groups. The new paradigm was of paramount importance for the development of mathematics: it foreshadowed the creation of abstract algebra in the works of Hilbert, Emil Artin, Emmy Noether, and mathematicians of their school.[8]

Groups with additional structure

An important elaboration of the concept of a group occurs if G is endowed with additional structure, notably, of a topological space, differentiable manifold, or algebraic variety. If the group operations m (multiplication) and i (inversion), are compatible with this structure, that is, they are continuous, smooth or regular (in the sense of algebraic geometry) maps, then G is a topological group, a Lie group, or an algebraic group.[2]

The presence of extra structure relates these types of groups with other mathematical disciplines and means that more tools are available in their study. Topological groups form a natural domain for abstract harmonic analysis, whereas Lie groups (frequently realized as transformation groups) are the mainstays of differential geometry and unitary representation theory. Certain classification questions that cannot be solved in general can be approached and resolved for special subclasses of groups. Thus, compact connected Lie groups have been completely classified. There is a fruitful relation between infinite abstract groups and topological groups: whenever a group Γ can be realized as a lattice in a topological group G, the geometry and analysis pertaining to G yield important results about Γ. A comparatively recent trend in the theory of finite groups exploits their connections with compact topological groups (profinite groups): for example, a single p-adic analytic group G has a family of quotients which are finite p-groups of various orders, and properties of G translate into the properties of its finite quotients.[9]

During the twentieth century, mathematicians investigated some aspects of the theory of finite groups in great depth, especially the local theory of finite groups and the theory of solvable and nilpotent groups. As a consequence, the complete classification of finite simple groups was achieved, meaning that all those simple groups from which all finite groups can be built are now known.[10]

| ISSN: 2395-7639 | www.ijmrsetm.com | Impact Factor: 7.580| A Monthly Double-Blind Peer Reviewed Journal |

| Volume 8, Issue 12, December 2021 |

| DOI: 10.15680/IJMRSETM.2021.0812009 |

During the second half of the twentieth century, mathematicians such as Chevalley and Steinberg also increased our understanding of finite analogs of classical groups, and other related groups. One such family of groups is the family of general linear groups over finite fields. Finite groups often occur when considering symmetry of mathematical or physical objects, when those objects admit just a finite number of structure-preserving transformations. The theory of Lie groups, which may be viewed as dealing with "continuous symmetry", is strongly influenced by the associated Weyl groups. These are finite groups generated by reflections which act on a finite-dimensional Euclidean space. The properties of finite groups can thus play a role in subjects such as theoretical physics and chemistry.[11]

III.DISCUSSION

A Lie group is a group that is also a differentiable manifold, with the property that the group operations are compatible with the smooth structure. Lie groups are named after Sophus Lie, who laid the foundations of the theory of continuous transformation groups. The term *groupes de Lie* first appeared in French in 1893 in the thesis of Lie's student Arthur Tresse, page 3.[5]

Lie groups represent the best-developed theory of continuous symmetry of mathematical objects and structures, which makes them indispensable tools for many parts of contemporary mathematics, as well as for modern theoretical physics. They provide a natural framework for analysing the continuous symmetries of differential equations (differential Galois theory), in much the same way as permutation groups are used in Galois theory for analysing the discrete symmetries of algebraic equations. An extension of Galois theory to the case of continuous symmetry groups was one of Lie's principal motivations.[12]

The axioms of a group formalize the essential aspects of symmetry. Symmetries form a group: they are closed because if you take a symmetry of an object, and then apply another symmetry, the result will still be a symmetry. The identity keeping the object fixed is always a symmetry of an object. Existence of inverses is guaranteed by undoing the symmetry and the associativity comes from the fact that symmetries are functions on a space, and composition of functions is associative.

Frucht's theorem says that every group is the symmetry group of some graph. So every abstract group is actually the symmetries of some explicit object.
The saying of "preserving the structure" of an object can be made precise by working in a category. Maps preserving the structure are then the morphisms, and the symmetry group is the automorphism group of the object in question.[13]
Applications of group theory abound. Almost all structures in abstract algebra are special cases of groups. Rings, for example, can be viewed as abelian groups (corresponding to addition) together with a second operation (corresponding to multiplication). Therefore, group theoretic arguments underlie large parts of the theory of those entities.

Galois theory uses groups to describe the symmetries of the roots of a polynomial (or more precisely the automorphisms of the algebras generated by these roots). The fundamental theorem of Galois theory provides a link between algebraic field extensions and group theory. It gives an effective criterion for the solvability of polynomial equations in terms of the solvability of the corresponding Galois group. For example, S_5, the symmetric group in 5 elements, is not solvable which implies that the general quintic equation cannot be solved by radicals in the way equations of lower degree can. The theory, being one of the historical roots of group theory, is still fruitfully applied to yield new results in areas such as class field theory.[14]

Algebraic topology is another domain which prominently associates groups to the objects the theory is interested in. There, groups are used to describe certain invariants of topological spaces. They are called "invariants" because they are defined in such a way that they do not change if the space is subjected to some deformation. For example, the fundamental group "counts" how many paths in the space are essentially different. The Poincaré conjecture, proved in 2002/2003 by Grigori Perelman, is a prominent application of this idea. The influence is not unidirectional, though. For example, algebraic topology makes use of Eilenberg–MacLane spaces which are spaces with prescribed homotopy groups. Similarly algebraic K-theory relies in a way on classifying spaces of groups. Finally, the name of the torsion subgroup of an infinite group shows the legacy of topology in group theory.[15]

Algebraic geometry likewise uses group theory in many ways. Abelian varieties have been introduced above. The presence of the group operation yields additional information which makes these varieties particularly accessible. They also often serve as a test for new conjectures. (For example the Hodge conjecture (in certain cases).) The one-dimensional case, namely elliptic curves is studied in particular detail. They are both theoretically and practically

| ISSN: 2395-7639 | www.ijmrsetm.com | Impact Factor: 7.580| A Monthly Double-Blind Peer Reviewed Journal |

| Volume 8, Issue 12, December 2021 |

| DOI: 10.15680/IJMRSETM.2021.0812009 |

intriguing.[8] In another direction, toric varieties are algebraic varieties acted on by a torus. Toroidal embeddings have recently led to advances in algebraic geometry, in particular resolution of singularities.[9]

Very large groups of prime order constructed in elliptic curve cryptography serve for public-key cryptography. Cryptographical methods of this kind benefit from the flexibility of the geometric objects, hence their group structures, together with the complicated structure of these groups, which make the discrete logarithm very hard to calculate. One of the earliest encryption protocols, Caesar's cipher, may also be interpreted as a (very easy) group operation. Most cryptographic schemes use groups in some way. In particular Diffie–Hellman key exchange uses finite cyclic groups. So the term group-based cryptography refers mostly to cryptographic protocols that use infinite nonabelian groups such as a braid group.[16]

IV.RESULTS

Some elementary **examples of groups** in mathematics are given :-

Consider three colored blocks (red, green, and blue), initially placed in the order RGB. Let a be the operation "swap the first block and the second block", and b be the operation "swap the second block and the third block".

We can write xy for the operation "first do y, then do x"; so that ab is the operation RGB → RBG → BRG, which could be described as "move the first two blocks one position to the right and put the third block into the first position". If we write e for "leave the blocks as they are" (the identity operation), then we can write the six permutations of the three blocks as follows:

- e : RGB → RGB
- a : RGB → GRB
- b : RGB → RBG
- ab : RGB → BRG
- ba : RGB → GBR
- aba : RGB → BGR

Note that aa has the effect RGB → GRB → RGB; so we can write $aa = e$. Similarly, $bb = (aba)(aba) = e$; $(ab)(ba) = (ba)(ab) = e$; so every element has an inverse.

By inspection, we can determine associativity and closure; note in particular that $(ba)b = bab = b(ab)$.

Since it is built up from the basic operations a and b, we say that the set $\{a, b\}$ *generates* this group. The group, called the *symmetric group* S_3, has order 6, and is non-abelian (since, for example, $ab \neq ba$).

A *translation* of the plane is a rigid movement of every point of the plane for a certain distance in a certain direction. For instance "move in the North-East direction for 2 miles" is a translation of the plane. Two translations such as a and b can be composed to form a new translation $a \circ b$ as follows: first follow the prescription of b, then that of a. For instance, if

a = "move North-East for 3 miles"

and

b = "move South-East for 4 miles"

then

$a \circ b$ = "move to bearing 8.13° for 5 miles" *(bearing is measured counterclockwise and from East)*

Or, if

a = "move to bearing 36.87° for 3 miles" *(bearing is measured counterclockwise and from East)*

and

b = "move to bearing 306.87° for 4 miles" *(bearing is measured counterclockwise and from East)*

then

$a \circ b$ = "move East for 5 miles"

(see Pythagorean theorem for why this is so, geometrically).

The set of all translations of the plane with composition as the operation forms a group:

1. If a and b are translations, then $a \circ b$ is also a translation.
2. Composition of translations is associative: $(a \circ b) \circ c = a \circ (b \circ c)$.
3. The identity element for this group is the translation with prescription "move zero miles in any direction".

4. The inverse of a translation is given by walking in the opposite direction for the same distance.

This is an abelian group and our first (nondiscrete) example of a Lie group: a group which is also a manifold.[17]
Groups are very important to describe the symmetry of objects, be they geometrical (like a tetrahedron) or algebraic (like a set of equations). As an example, we consider a glass square of a certain thickness (with a letter "F" written on it, just to make the different positions distinguishable).

In order to describe its symmetry, we form the set of all those rigid movements of the square that don't make a visible difference (except the "F"). For instance, if an object turned 90° clockwise still looks the same, the movement is one element of the set, for instance *a*. We could also flip it around a vertical axis so that its bottom surface becomes its top surface, while the left edge becomes the right edge. Again, after performing this movement, the glass square looks the same, so this is also an element of our set and we call it *b*. The movement that does nothing is denoted by *e*.[18]
Given two such movements x and y, it is possible to define the composition $x \circ y$ as above: first the movement y is performed, followed by the movement x. The result will leave the slab looking like before.
The point is that the set of all those movements, with composition as the operation, forms a group. This group is the most concise description of the square's symmetry. Chemists use symmetry groups of this type to describe the symmetry of crystals and molecules.

V.CONCLUSIONS

Let's investigate our square's symmetry group some more. Right now, we have the elements a, b and e, but we can easily form more: for instance $a \circ a$, also written as a^2, is a 180° degree turn. a^3 is a 270° clockwise rotation (or a 90° counter-clockwise rotation). We also see that $b^2 = e$ and also $a^4 = e$. Here's an interesting one: what does $a \circ b$ do? First flip horizontally, then rotate. Try to visualize that $a \circ b = b \circ a^3$. Also, $a^2 \circ b$ is a vertical flip and is equal to $b \circ a^2$.
We say that elements a and b generate the group.
This group of order 8 has the following Cayley table:

∘	**e**	**b**	**a**	**a^2**	**a^3**	**ab**	**a^2b**	**a^3b**
e	**e**	b	a	a^2	a^3	ab	a^2b	a^3b
b	b	**e**	a^3b	a^2b	ab	a^3	a^2	a
a	a	ab	a^2	a^3	**e**	a^2b	a^3b	b
a^2	a^2	a^2b	a^3	**e**	a	a^3b	b	ab
a^3	a^3	a^3b	**e**	a	a^2	b	ab	a^2b
ab	ab	a	b	a^3b	a^2b	**e**	a^3	a^2
a^2b	a^2b	a^2	ab	b	a^3b	a	**e**	a^3
a^3b	a^3b	a^3	a^2b	ab	b	a^2	a	**e**

For any two elements in the group, the table records what their composition is. Here we wrote "a^3b" as a shorthand for $a^3 \circ b$.

In mathematics this group is known as the **dihedral group** of order 8, and is either denoted Dih_4, D_4 or D_8, depending on the convention. This was an example of a non-abelian group: the operation ∘ here is not commutative, which can be seen from the table; the table is not symmetrical about the main diagonal.[19]

This version of the Cayley table shows that this group has one normal subgroup shown with a red background. In this table r means rotations, and f means flips. Because the subgroup is normal, the left coset is the same as the right coset.
The free group with two generators a and b consists of all finite strings/words that can be formed from the four symbols a, a^{-1}, b and b^{-1} such that no a appears directly next to an a^{-1} and no b appears directly next to a b^{-1}. Two such strings can be concatenated and converted into a string of this type by repeatedly replacing the "forbidden" substrings with the empty string. For instance: "$abab^{-1}a^{-1}$" concatenated with "$abab^{-1}a$" yields "$abab^{-1}a^{-1}abab^{-1}a$", which gets reduced to "$abaab^{-1}a$". One can check that the set of those strings with this operation forms a group with the empty string ε := "" being the identity element (Usually the quotation marks are left off; this is why the symbol ε is required).[20]

| ISSN: 2395-7639 | www.ijmrsetm.com | Impact Factor: 7.580| A Monthly Double-Blind Peer Reviewed Journal |

| Volume 8, Issue 12, December 2021 |

| DOI: 10.15680/IJMRSETM.2021.0812009 |

This is another infinite non-abelian group.
Free groups are important in algebraic topology; the free group in two generators is also used for a proof of the Banach–Tarski paradox[21]

Group table of D_4

	e	**r_1**	**r_2**	**r_3**	**f_v**	**f_h**	**f_d**	**f_c**
e	e	r_1	r_2	r_3	f_v	f_h	f_d	f_c
r_1	r_1	r_2	r_3	e	f_c	f_d	f_v	f_h
r_2	r_2	r_3	e	r_1	f_h	f_v	f_c	f_d
r_3	r_3	e	r_1	r_2	f_d	f_c	f_h	f_v
f_v	f_v	f_d	f_h	f_c	e	r_2	r_1	r_3
f_h	f_h	f_c	f_v	f_d	r_2	e	r_3	r_1
f_d	f_d	f_h	f_c	f_v	r_3	r_1	e	r_2
f_c	f_c	f_v	f_d	f_h	r_1	r_3	r_2	e

The elements e, r_1, r_2, and r_3 form a subgroup, highlighted in red (upper left region). A left and right coset of this subgroup is highlighted in green (in the last row) and yellow (last column), respectively.

If *n* is some positive integer, we can consider the set of all invertible *n* by *n* matrices with real number components, say. This is a group with matrix multiplication as the operation. It is called the **general linear group**, and denoted $GL_n(\mathbf{R})$ or $GL(n, \mathbf{R})$ (where **R** is the set of real numbers). Geometrically, it contains all combinations of rotations, reflections, dilations and skew transformations of *n*-dimensional Euclidean space that fix a given point (the origin).
If we restrict ourselves to matrices with determinant 1, then we get another group, the **special linear group**, $SL_n(\mathbf{R})$ or $SL(n, \mathbf{R})$. Geometrically, this consists of all the elements of $GL_n(\mathbf{R})$ that preserve both orientation and volume of the various geometric solids in Euclidean space.

If instead we restrict ourselves to orthogonal matrices, then we get the **orthogonal group** $O_n(\mathbf{R})$ or $O(n, \mathbf{R})$. Geometrically, this consists of all combinations of rotations and reflections that fix the origin. These are precisely the transformations which preserve lengths and angles.

Finally, if we impose both restrictions, then we get the **special orthogonal group** $SO_n(\mathbf{R})$ or $SO(n, \mathbf{R})$, which consists of rotations only.[22]

These groups are our first examples of infinite non-abelian groups. They are also happen to be Lie groups. In fact, most of the important Lie groups (but not all) can be expressed as matrix groups.
If this idea is generalised to matrices with complex numbers as entries, then we get further useful Lie groups, such as the unitary group U(*n*). We can also consider matrices with quaternions as entries; in this case, there is no well-defined notion of a determinant (and thus no good way to define a quaternionic "volume"), but we can still define a group analogous to the orthogonal group, the **symplectic group** Sp(*n*).

Furthermore, the idea can be treated purely algebraically with matrices over any field, but then the groups are not Lie groups.

| ISSN: 2395-7639 | www.ijmrsetm.com | Impact Factor: 7.580| A Monthly Double-Blind Peer Reviewed Journal |

| Volume 8, Issue 12, December 2021 |

| DOI: 10.15680/IJMRSETM.2021.0812009 |

For example, we have the general linear groups over finite fields. The group theorist J. L. Alperin has written that "The typical example of a finite group is GL(*n*, *q*), the general linear group of *n* dimensions over the field with *q* elements. The student who is introduced to the subject with other examples is being completely misled."[23]

REFERENCES

1. Elwes, Richard (December 2006), "An enormous theorem: the classification of finite simple groups", Plus Magazine (41), archived from the original on 2009-02-02, retrieved 2011-12-20
2. ^ This process of imposing extra structure has been formalized through the notion of a group object in a suitable category. Thus Lie groups are group objects in the category of differentiable manifolds and affine algebraic groups are group objects in the category of affine algebraic varieties.
3. ^ Such as group cohomology or equivariant K-theory.
4. ^ In particular, if the representation is faithful.
5. ^ Arthur Tresse (1893), "Sur les invariants différentiels des groupes continus de transformations", Acta Mathematica, **18**: 1–88, doi:10.1007/bf02418270
6. ^ Schupp & Lyndon 2001
7. ^ La Harpe 2000
8. ^ See the Birch and Swinnerton-Dyer conjecture, one of the millennium problems
9. ^ Abramovich, Dan; Karu, Kalle; Matsuki, Kenji; Wlodarczyk, Jaroslaw (2002), "Torification and factorization of birational maps", Journal of the American Mathematical Society, **15** (3): 531–572, arXiv:math/9904135, doi:10.1090/S0894-0347-02-00396-X, MR 1896232, S2CID 18211120
10. ^ Lenz, Reiner (1990), Group theoretical methods in image processing, Lecture Notes in Computer Science, vol. 413, Berlin, New York: Springer-Verlag, doi:10.1007/3-540-52290-5, ISBN 978-0-387-52290-6, S2CID 2738874
11. ^ Norbert Wiener, Cybernetics: Or Control and Communication in the Animal and the Machine, ISBN 978-0262730099, Ch 2
12. Alperin, Jonathan L. (1984). "Book Review: Finite groups". Bulletin of the American Mathematical Society. New Series. **10**: 121–124. doi:10.1090/S0273-0979-1984-15210-8.
13. Borel, Armand (1991), Linear algebraic groups, Graduate Texts in Mathematics, vol. 126 (2nd ed.), Berlin, New York: Springer-Verlag, doi:10.1007/978-1-4612-0941-6, ISBN 978-0-387-97370-8, MR 1102012
14. Carter, Nathan C. (2009), Visual group theory, Classroom Resource Materials Series, Mathematical Association of America, ISBN 978-0-88385-757-1, MR 2504193
15. Cannon, John J. (1969), "Computers in group theory: A survey", Communications of the ACM, **12**: 3–12, doi:10.1145/362835.362837, MR 0290613, S2CID 18226463
16. Frucht, R. (1939), "Herstellung von Graphen mit vorgegebener abstrakter Gruppe", Compositio Mathematica, **6**: 239–50, ISSN 0010-437X, archived from the original on 2008-12-01
17. Golubitsky, Martin; Stewart, Ian (2006), "Nonlinear dynamics of networks: the groupoid formalism", Bull. Amer. Math. Soc. (N.S.), **43** (3): 305–364, doi:10.1090/S0273-0979-06-01108-6, MR 2223010 Shows the advantage of generalising from group to groupoid.
18. Judson, Thomas W. (1997), Abstract Algebra: Theory and Applications An introductory undergraduate text in the spirit of texts by Gallian or Herstein, covering groups, rings, integral domains, fields and Galois theory. Free downloadable PDF with open-source GFDL license.
19. Kleiner, Israel (1986), "The evolution of group theory: a brief survey", Mathematics Magazine, **59** (4): 195–215, doi:10.2307/2690312, ISSN 0025-570X, JSTOR 2690312, MR 0863090
20. La Harpe, Pierre de (2000), Topics in geometric group theory, University of Chicago Press, ISBN 978-0-226-31721-2
21. Livio, M. (2005), The Equation That Couldn't Be Solved: How Mathematical Genius Discovered the Language of Symmetry, Simon & Schuster, ISBN 0-7432-5820-7 Conveys the practical value of group theory by explaining how it points to symmetries in physics and other sciences.
22. Mumford, David (1970), Abelian varieties, Oxford University Press, ISBN 978-0-19-560528-0, OCLC 138290
23. Ronan M., 2006. Symmetry and the Monster. Oxford University Press. ISBN 0-19-280722-6. For lay readers. Describes the quest to find the basic building blocks for finite groups.